HW1 **NAME: ....** **due Wednesday, 1/17/2018**

Homework contains "warm up" problems, which you do not need to write if you know how to solve them. Similar calculations will come up as parts of other problems in the future. If in doubt (whether you can solve correctly or not), please write your solution, and I will check whether it is correct. For example, this assignment contains 3 problems (out of 9) that are necessary to submit; they are marked by **{*}**.

1. {"warm up" problem} Using the Euler formula, derive the trigonometric identity

$$\cos(3\alpha) = 4\cos^3\alpha - 3\cos\alpha \qquad \text{and} \qquad \sin(3\alpha) = -4\sin^3\alpha + 3\sin\alpha$$

{*Suggestion: Take the real and imaginary part of the identity* $e^{3i\alpha} = \left(e^{i\alpha}\right)^3$.}

2. {"warm up" problem} Using the Euler formula, derive the trigonometric identity

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \text{and} \qquad \sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

{*Suggestion: Take the real and imaginary part of the identity* $e^{i\alpha} + e^{i\beta} = e^{i\frac{\alpha+\beta}{2}}\left(e^{i\frac{\alpha-\beta}{2}} + e^{-i\frac{\alpha-\beta}{2}}\right)$*; here the expression in paranthesis is real.*}

3. **{*}** Show that

$$\sin x + \sin 2x + \ldots + \sin Nx = \frac{\sin\frac{N}{2}x \cdot \sin\frac{N+1}{2}x}{\sin\frac{x}{2}}$$

{*Hint:* $\sin\alpha = \mathrm{Im}(e^{i\alpha})$. *Geometric sequence.*}

4. {“warm up” problem} Find all values of 1^i.

5. {“warm up” problem} Solve the equation $\cos z = 3$.

6. **{*}** Find all values of $\arctan(3i)$. {*Hint:* $w = \arctan 3i \quad \overset{\text{def}}{\Leftrightarrow} \quad \tan w = 3i$.}

7. {"warm up" problem} A function $w(z)$ of a complex variable $z = x + iy$ takes only real values.
   Can it be dierentiable (in complex sense) at some points?
   Can it be analytic?

8. {"warm up" problem}

   (a) For the function $u(x, y) = x + 4y$, find the function $v(x, y)$ such that $w = u + iv$ is an analytic function of the variable $z = x + iy$.

   (b) For $v(x, y) = (1 + x)y$, find $u(x, y)$ such that $w = u + iv$ is an analytic function of the variable $z = x + iy$.

   (c) Consider the function $u(x, y) = x^2$. Does there exist a function $v(x, y)$ such that $w = u + iv$ is an analytic function of the variable $z = x + iy$.

9. **{*}**

   (a) Show: If a function $w(z) = u(x, y) + iv(x, y)$ is analytic in a domain $D \in \mathbb{C} \quad (z = x + iy)$, then $u(x, y)$ and $v(x, y)$ are harmonic functions in that domain.

   (b) Show: If a function $u(x, y)$ is harmonic in a *simply connected* domain $D \in \mathbb{R}^2$, then there is a function $v(x, y)$, $[(x, y) \in D]$, such that $u(x, y) + iv(x, y) = w(z)$ is analytic in D.

   (c) Would the last statement hold without the requirement of D being *simply connected*?