

Math 3160

Applied Complex Variables

Spring 2008

NAME:.....

Final

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1. Find all complex roots of the equation $\cos z = 2$.

2. Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate the integral

$$\int_C \frac{\cos z}{z(z^2 + 8)} dz.$$

3. Give two Laurent series expansions *in powers of* z for the function

$$f(z) = \frac{1}{z^2(3i - z)},$$

and specify the regions in which those expansions are valid.

4. Evaluate the following integral

$$\int_{-\infty}^{+\infty} \frac{x \sin ax}{x^4 + 4} dx$$

(where a is a positive parameter, $a > 0$).

5. Evaluate the following integral

$$\int_0^{2\pi} \frac{d\theta}{1 + a \cos \theta}$$

(where a is a parameter, $-1 < a < 1$).

6. Find the image of the semi-infinite strip $x > 0, 0 < y < 1$ when $w = i/z$. Sketch the strip and its image ($z = x + iy, w = u + iv$).

7. Find a conformal transformation that maps two non-concentric circles

$$|z| = 1 \quad \text{and} \quad |z - 9/28| = 5/28$$

into concentric circles. Sketch the two circles before and after the transformation.