M1220-2 **Diagnostic Quiz** **Spring 2005**

Quiz Scores (out of 10): n = 133; Mean = 5.7; median = 5.5

1. (4 pts) Let $f(x) = [\sin x + 1]^2$. Find $f'(\pi)$ and use your answer to find the equation of the tangent to the curve at $(\pi, 1)$.

$f'(x) = 2[\sin x + 1]\cos x$
$f'(\pi) = 2[\sin \pi + 1]\cos \pi = 2[0 + 1](-1) = -2$

Equation of tangent at $(\pi, 1)$: The slope of the tangent to the curve at $(\pi, 1)$ is $f'(\pi)$ so set, $y = mx + b = -2x + b$. Since $(\pi, 1)$ is a point on the curve, we can substitute $x = \pi, y = 1$ into this equation to find b. Then $1 = -2\pi + b$, so $b = 1 + 2\pi$. Then equation of the tangent at $(\pi, 1)$ is: $y = -2x + (1 + 2\pi$.

2. (4 pts) **Integrals**

i)Evaluate: $\int_0^1 \frac{t}{(t^2+1)^2} dt$

Let $u = t^2 + 1$. Then $du = 2tdt$. So, $\int_0^1 \frac{t}{(t^2+1)^2} dt = 1/2 \int_1^2 \frac{1}{u^2} du = 1/2[-u^{-1}|_1^2] = 1/4$

Note the change in the integration limits when we use the substitution, $u = t^2 + 1$. When $t = 0, u = 1$, and when $t = 1, u = 2$.

ii) If $F(x) = \int_0^x \frac{t}{(t^2+1)^2} dt$, what is $F'(x)$?

Since $f(t) = \frac{t}{(t^2+1)^2}$ is continuous for all real numbers, by the First Fundamental Theorem of Calculus, $F'(x) = f(x) = \frac{x}{(x^2+1)^2}$.

3.(2 pts) Let $h(x) = f(x) - g(x)$ where f and g are differentiable functions on the real numbers. Suppose $f(1) = g(1)$, and that $f'(x) > g'(x)$ for all x in [1, 10]. Is $h(x)$ positive on the open interval (1,10)? Choose one of the following:

Since $f(1) = g(1), h(1) = f(1) - g(1) = 0$. Also, since $f'(x) > g'(x)$ on [1, 10], $h'(x) = f'(x) - g'(x) > 0$ on [1, 10] and so is strictly increasing on that interval. If $h(1) = 0$, then $h(x) > 0$ for all x in (1, 10). So the answer is: Yes.