M1220-2 **Quiz 9 - Homework Quiz** **Spring 2005**

Quiz Scores (out of 10): n = 121; mean = 7.8

25th percentile = 7 ; median (50th percentile) = 8 ; 75th percentile = 9.5

1. (6 pts) State whether the following series are absolutely convergent, conditionally convergent, or divergent. Justify your answer by listing the test used and showing the test results.

i.

$$\sum_{n=1}^{+\infty} \frac{n(-3)^n}{4^{n-1}}$$

Note that $\lim_{n\to+\infty} |a_{n+1}/a_n| = \lim_{n\to+\infty} \frac{(n+1)3^{n+1}}{4^n} \frac{4^{n-1}}{n3^n} = \lim_{n\to+\infty} \frac{n+1}{n} \frac{3}{4} = \frac{3}{4} < 1$. So, by the Ratio Test, this series converges absolutely.

ii.

$$\sum_{n=1}^{+\infty} \frac{n!}{(-5)^n}$$

Note that $\lim_{n\to+\infty} |a_{n+1}/a_n| = \lim_{n\to+\infty} \frac{(n+1)!}{5^{n+1}} \frac{5^n}{n!} = \lim_{n\to+\infty} \frac{(n+1)!}{n!} \frac{5^n}{5^{n+1}} =$ $\lim_{n\to+\infty} \frac{n+1}{5} = +\infty$. So, by the Ratio Test, $\sum_{n=1}^{+\infty} \frac{n!}{(-5)^n}$ diverges.

iii.

$$\sum_{n=1}^{+\infty} (-1)^n \frac{n}{3+n^2}$$

Note that $\lim_{n\to+\infty} |a_{n+1}/a_n|$

$= \lim_{n\to+\infty} \frac{n+1}{3+(n+1)^2} \frac{3+n^2}{n} = 1$

so the Ratio Test is inclusive. However, the series, $\sum \frac{n}{3+n^2}$ is similar to the series, $\sum 1/n$ which diverges. Since

$\lim_{n\to+\infty} \left[\frac{n}{3+n^2}\right] / \frac{1}{n} = 1,$

by the Limit Comparison Test, the series $\sum \frac{n}{3+n^2}$ also diverges. In that case, the given series cannot converge absolutely.

However, since the terms of the given series, ignoring their sign, are decreasing and since $\lim_{n\to+\infty} \frac{n}{3+n^2} = 0$, by the Alternating Series Test, the series $\sum_{n=1}^{+\infty} (-1)^n \frac{n}{3+n^2}$ converges conditionally.

2. (2 pts) Write out the first 4 terms in the power series given below. Then find the convergence set for the power series and the radius of convergence.

$$\sum_{n=o}^{+\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$$

The first four terms are: $1 - \frac{3x}{\sqrt{2}} + \frac{9x^2}{\sqrt{3}} - \frac{27x3}{\sqrt{4}}$

$\lim_{n\to+\infty} |a_{n+1}/a_n| = \lim_{n\to+\infty} |\frac{3^{n+1}x^{n+1}}{\sqrt{n+2}} \frac{\sqrt{n+1}}{3^n x^n}| = \lim_{n\to+\infty} |3x\frac{\sqrt{n+1}}{\sqrt{n+2}}| = |3x|$. But $|3x| <$ 1when $-1/3 < x < 1/3$ so the power series converges when $-1/3 < x < 1/3$. At the endpoints of the interval: when $x = 1/3$, the above series becomes $\sum \frac{(-1)^n}{\sqrt{n+1}}$ which converges by the Alternating Series Test; when $x = -1/3$, the above series becomes $\sum \frac{1}{\sqrt{n+1}}$which diverges as it is a p-series with $p \leq 1$. So, the convergence set for this series is $(-1/3, 1/3]$ and the radius of convergence is $R = 1/3$.

3. (2 pts) Write out the first 4 terms in the power series given below. Then find the convergence set for the power series and the radius of convergence.

$$\sum_{n=1}^{+\infty} \frac{(x-2)^n}{n^n}$$

The first four terms are: $(x-2) + (x-2)^2/2^2 + (x-2)^3/3^3 + (x-2)^4/4^4$.

$\lim_{n\to+\infty} |a_{n+1}/a_n| = \lim_{n\to+\infty} |\frac{(x-2)^{n+1}}{(n+1)^{n+1}} \frac{n^n}{(x-2)^n}| = \lim_{n\to+\infty} |\frac{x-2}{n+1} \left[\frac{n}{n+1}\right]^n| =$ $|x-2| \lim_{n\to+\infty} \frac{1}{n+1}(\frac{n}{n+1})^n$. Since $\lim_{n\to+\infty}(\frac{n}{n+1})^n = 1/e$ and $\lim_{n\to+\infty} \frac{1}{n+1} = 0$, $\lim_{n\to+\infty}(\frac{1}{n+1}(\frac{n}{n+1})^n = 0$. So, the convergence set for this series is $(-\infty, +\infty)$ and the radius of convergence is $R = +\infty$.