M1220-2 **Quiz 8** **Spring 2005**

Name:____

PLEASE PRINT

Instructions: There are three questions. One is on the back. Show your work on each question.

1. (2 pts) Find the sum of the following convergent geometric series. **Note that the series begins at k = 3.**

$$\sum_{k=3}^{+\infty} \left(\frac{-2}{3}\right)^{k-1}$$

$$\sum_{k=3}^{+\infty} \left(\frac{-2}{3}\right)^{k-1} = (-2/3)^2 + (-2/3)^2\left(\frac{-2}{3}\right) + (-2/3)^2\left(\frac{-2}{3}\right)^2 + (-2/3)^2\left(\frac{-2}{3}\right)^3 + ...$$

. So, $a = (-2/3)^2$ and $r = \frac{-2}{3}$ and the sum of the series is

$$\frac{a}{1-r} = \frac{(-2/3)^2}{1-(-2/3)} = \frac{4}{15}$$

.

2. (2 pt) P-series:
Give examples of two P-series, one that converges and one that diverges.

Convergent P-series: Any series of the form $\sum_{k=1}^{+\infty} \frac{1}{k^p}$ where $p > 1$.

Divergent P-series: Any series of the form $\sum_{k=1}^{+\infty} \frac{1}{k^p}$ where $p \leq 1$.

3. (6 pts) Do the following series converge or diverge? Justify your conclusion by stating the test used and showing the results of the test. For example:
$\sum_{k=1}^{+\infty} ke^{-k^2}$ converges by the Integral Test since for $u = x^2$, $\int_1^{+\infty} xe^{-x^2}dx = \frac{1}{2}\int_1^{+\infty} e^{-u}du$ which converges.
$\sum_{k=1}^{+\infty} \left(\frac{7}{5}\right)^k$ diverges since it is a geometric series with $|r| > 1$.

i)

$$\sum_{k=2}^{+\infty} \frac{1}{k\sqrt{\ln k}}$$

This series diverges by the Integral test. Note that if we let $f(x) = \frac{1}{x\sqrt{\ln x}}$, then f is positive, continuous, and nonincreasing on the interval $[1, +\infty)$ and that letting $u = \ln x$we have, $\int_2^{+\infty} \frac{1}{x\sqrt{\ln x}}dx = \int_{ln2}^{+\infty} \frac{du}{u^{1/2}}$ which diverges since this is an integral over $[a, +\infty), a > 0$ that is of the form $\int \frac{1}{u^p}du$ with $p = 1/2 < 1$.

ii)

$$\sum_{k=1}^{+\infty} \frac{\sin^2(k)}{2^k}$$

This integral converges by the Comparison Test. Note that for all $k, 0 \leq \frac{\sin^2(k)}{2^k} \leq \frac{1}{2^k}$ since $\sin^2(k) \leq 1$. But $\sum \frac{1}{2^k}$ is a convergent geometric series since $|r = 1/2| < 1$. So, the series $\sum_{k=1}^{+\infty} \frac{\sin^2(k)}{2^k}$ is also convergent.

iii)

$$\sum_{k=1}^{+\infty} k\left(\frac{1}{3}\right)^k \quad \text{Note this is not a geometric series!}$$

This series is convergent by the Ratio Test. Note that $\lim_{k\to+\infty} \frac{a_{k+1}}{a_k}$
$= \lim_{k\to+\infty} \left[\frac{k+1}{3^{k+1}}\frac{3^k}{k}\right] = \lim_{k\to+\infty} \left[\frac{1}{3}\frac{k+1}{k}\right] = \frac{1}{3} < 1$. So by the Ratio Test the series converges.