M1220-2 **Quiz 7** **Spring 2005**

Quiz Scores (out of 10): n = 129; mean = 7.33
25th percentile = 6; median (50th percentile) = 7.5; 75th percentile = 9.0

1. (6 pts) Determine whether the following sequences converge or diverge. If the sequence converges, give the limit of the sequence. If the sequence diverges, state why.

Comment: Note that in this problem you are asked whether a **sequence** converges, not whether a **series** converges.

i)

$$a_n = \left(\frac{-e}{2\pi}\right)^n$$

Note that this is an example of a sequence of the form, $\{r^n\}$ where $r = \frac{-e}{2\pi}$. Since $|\frac{-e}{2\pi}| < 1$, the sequence $\{a_n\}$ converges and

$$\lim_{n\to+\infty} \left(\frac{-e}{2\pi}\right)^n = 0.$$

ii)

$$a_n = \frac{\cos n\pi}{n}$$

Note that $\cos n\pi$ alternates between $+1$ and -1 so the sequence is really $\{(-1)^n(1/n)\}$. This sequence oscillates about zero but because $1/n \to 0$ as $n \to +\infty$, the sequence $\frac{\cos n\pi}{n}$ converges to zero.

iii)

$$a_n = (-1)^n \frac{3n}{2n+1}$$

As in ii) this sequence oscillates about zero but since $\frac{3n}{2n+1} \to 3/2$ as $n \to +\infty$, the odd terms in the sequence will approach $+3/2$ and the even terms in the sequence will approach -3/2. So, this sequence diverges.

2. (4 pts) Determine whether the following infinite series converge or diverge. If a series converges, find the sum of the series where possible. If the series diverges briefly state why.

i)

$$\sum_{k=1}^{+\infty}(2/5)^{k-1}$$

This is the geometric series, $1+2/5+(2/5)^2+(2/5)^3+\ldots$, with $a=1, r=2/5$. Since $|r|<1$, this series converges and its sum is $\frac{a}{1-r}=\frac{1}{1-2/5}=5/3$. This means that the sequence of partial sums has limit $=5/3$ and that by our definition,

$$\sum_{k=1}^{+\infty}(2/5)^{k-1}=5/3$$

.

ii)

$$\sum_{k=1}^{+\infty}\frac{e^k}{3k}$$

This series will diverge because as $k\to+\infty$, the terms in the sequence do not approach zero. (the nth term test for divergence)