M1220-2 **Quiz 5** **Spring 2005**

Quiz Scores (out of 10): n = 136; mean = 7.2
25th percentile = 5.7; median (50th percentile) = 7.5; 75th percentile = 9.0

1. (2.5 pts) Evaluate the following indefinite integral:

$$\int x^3 \ln(x^2) dx$$

If we let $u = \ln(x^2), dv = x^3 dx$, then $du = \frac{2}{x}dx, v = \frac{x^4}{4}$ and the above integral is of the form $\int u dv$. Then using integration by parts we have

$$\int x^3 \ln(x^2) dx = \frac{x^4}{4}\ln(x^2) - \int \frac{x^3}{2} dx = \frac{x^4}{4}\ln(x^2) - \frac{x^4}{8} + C$$

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Comments: We choose $u = \ln(x^2)$ because we can easily differentiate this expression. Note that you can also choose $u = x^2, du = 2x dx$ so that your integral becomes

$$\int x^3 \ln(x^2) dx = \frac{1}{2}\int x^2 \ln(x^2)(2x) dx = \frac{1}{2}\int u \ln(u) du$$

and here one uses integration by parts.

2. (2.5 pts) Evaluate the following indefinite integral:

$$\int \frac{1}{\sqrt{4 - x^2}} dx$$

Letting $x = 2\sin\theta$ we have $dx = 2\cos\theta d\theta$. So,

$$\int \frac{1}{\sqrt{4 - x^2}} dx = \int \frac{2\cos\theta}{\sqrt{4 - 4\sin^2\theta}} d\theta = \int \frac{2\cos\theta}{2\cos\theta} d\theta = \int d\theta = \theta + C$$

. Since $\sin\theta = x/2$, we have

$$\int \frac{1}{\sqrt{4-x^2}}dx = \arcsin(x/2) + C$$

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Comments: One might also recognise this as an arcsin derivative initially by writing $\sqrt{4-x^2} = 2\sqrt{1-(\frac{x}{2})^2}$. Then letting $u = \frac{x}{2}, du = \frac{dx}{2}$ we have

$$\int \frac{1}{\sqrt{4-x^2}}dx = \int \frac{du}{\sqrt{1-u^2}} = \arcsin(u) + C = \arcsin(\frac{x}{2}) + C$$

3. (2.5 pts) Evaluate the following indefinite integral:

$$\int \frac{x+5}{x^2+x-2}dx$$

Factoring x^2+x-2 into $(x-1)(x+2)$ we set the fraction $\frac{x+5}{x^2+x-2} = \frac{x+5}{(x-1)(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x+2)}$. Since $\frac{A}{(x-1)} + \frac{B}{(x+2)} = \frac{(A+B)x+(2A-B)}{(x-1)(x+2)}$, we have the two equations, $(A+B) = 1, (2A-B) = 5$. Solving these equations we have $A = 2, B = -1$. Then,

$$\int \frac{x+5}{x^2+x-2}dx = \int \frac{2}{(x-1)}dx + \int \frac{-1}{(x+2)}dx = 2\ln(|x-1|) - \ln(|x+2|) + C$$

. Then

$$\int \frac{x^2+2x+3}{x^2+x-2}dx = 2\ln(|x-1|) - \ln(|x+2|) + C$$

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4. (2.5 pts) Evaluate the following limit:

$$\lim_{x\to 0}\left[\frac{1}{x} - \frac{1}{\sin x}\right]$$

$$\lim_{x\to 0}\left[\frac{1}{x} - \frac{1}{\sin x}\right] = \lim_{x\to 0}\left[\frac{\sin x - x}{x\sin x}\right] = (0/0\text{form})$$

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Applying L'Hopital's Rule,

$$\lim_{x\to 0}\left[\frac{\sin x - x}{x\sin x}\right] = \lim_{x\to 0}\left[\frac{\cos x - 1}{x\cos x + \sin x}\right] = (0/0\text{form})$$

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Applying L'Hopital's Rule again, now to the last limit, we have,

$$\lim_{x\to 0}\left[\frac{\cos x - 1}{x\cos x + \sin x}\right] = \lim_{x\to 0}\left[\frac{-\sin x}{-x\sin x + \cos x + \cos x}\right] = 0/2 = 0$$

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