M1220-2 **Quiz 4** **Spring 2005**

Quiz Scores (out of 10): n = 126; mean = 4.36

25th percentile = 2; median (50th percentile) = 4.0; 75th percentile = 6.38

1. (2.5 pts)

$$\int_{-1}^{1} x\sqrt{x+1}dx$$

If we let $u = x+1$, then $du = dx$ and $x = u-1$. So, when $x = -1, u = 0$, and when $x = 1, u = 2$. Then we have,

$$\int_{-1}^{1} x\sqrt{x+1}dx = \int_{o}^{2} (u-1)u^{1/2}dx = \left[\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3}\right]|_0^2 = \frac{2\sqrt{32}}{5} - \frac{2\sqrt{8}}{3}$$

Comments: There are several other approaches one might use here. You could use integration by parts, letting $u = x, dv = \sqrt{x+1}$. Then you have, $uv - \int vdu = x(2/3)(x+1)^{3/2} - \int (3/2)(x+1)^{3/2}dx$. Or, you could let $u = \sqrt{x+1}$ so that $u^2 = x+1, x = u^2-1, dx = 2udu$. Then your integral becomes, $\int (u^2-1)u(2u)du = 2\int (u^4-u^2)du$.

2. (2.5 pts)

$$\int \frac{1}{x^2+2x+5}dx$$

Recognizing that $x^2+2x+5 = (x+1)^2+4 = 4\left[(\frac{x+1}{2})^2+1\right]$, we can use either of the following techniques:

Method 1: letting $u = \frac{x+1}{2}, du = (x/2)dx$ we have:

$$\int \frac{1}{x^2+2x+5}dx = \frac{1}{2}\int \frac{1/2}{\left[(\frac{x+1}{2})^2+1\right]}dx = \frac{1}{2}\int \frac{du}{u^2+1} = \frac{1}{2}\arctan(u) + C$$

. Then replacing u by $\frac{x+1}{2}$ we have

$$\int \frac{1}{x^2+2x+5}dx = \frac{1}{2}\text{arctan}(\frac{x+1}{2}) + C$$

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Method 2: If we let $\frac{x+1}{2} = \tan(\theta), -\pi/2 \le \theta \le \pi/2$, then $dx = 2\text{sec}^2(\theta)d\theta$. So we have,

$$\int \frac{1}{x^2+2x+5}dx = \frac{1}{2}\int \frac{2\text{sec}^2(\theta)}{\tan^2(\theta)+1}d\theta = \int \frac{\text{sec}^2(\theta)}{\text{sec}^2(\theta)}d\theta = \int d\theta = \theta + C.$$

Since $\frac{x+1}{2} = \tan(\theta)$, solving for θ we have $\theta = \text{arctan}(\frac{x+1}{2})$ and again

$$\int \frac{1}{x^2+2x+5}dx = \frac{1}{2}\text{arctan}(\frac{x+1}{2}) + C$$

.

3. (2.5 pts)

$$\int_0^{\pi/2} \sin^2(x)\cos^3(x)dx$$

$$\int_0^{\pi/2} \sin^2(x)\cos^3(x)dx = \int_0^{\pi/2} \sin^2(x)\cos^2(x)\cos(x)dx$$

. Replacing $\cos^2(x)$ with $(1-\sin^2(x))$ and letting $u = \sin(x), du = \cos(x)dx$ in the last integral we have

$$\int_0^{\pi/2} \sin^2(x)\cos^3(x)dx = \int_0^1 (u^2-u^4)du = \left[\frac{u^3}{3} - \frac{u^5}{5}\right]|_0^1 = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

.

Comments: Whenever you have an odd power, pull out one of those functions to the odd power to serve as your du. Note that if you replaced $\sin^2(x)$ by $(1-\cos^2(x)$, then the above integral becomes $\int \cos^3(x) - \cos^5(x)dx$ so

you still wind up with odd powers. Trying to use the half-angle formula, $sin^2(x) = \frac{1-\cos(2x)}{2}$ is not so useful because you now have a $\cos(\mathbf{2}x)$ and a $\cos(x)$ to deal with.
bigskip
4. (2.5 pts)

$$\int x \arctan(x) dx$$

Using integration by parts, let $u = \arctan(x), dv = xdx$. Then $du = \frac{1}{x^2+1}dx$ and $v = \frac{x^2}{2}$. So the above integral equals,

$$\frac{x^2}{2}\arctan(x) - \frac{1}{2}\int \frac{x^2}{x^2+1}dx$$

. Dividing (x^2+1) into x^2 we have, $\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$. Then

$$\frac{x^2}{2}\arctan(x) - \frac{1}{2}\int \frac{x^2}{x^2+1}dx = \frac{x^2}{2}\arctan(x) - \frac{1}{2}\left[x - \arctan(x)\right] + C$$

. So, the

$$\int x \arctan(x) dx = \frac{x^2}{2}\arctan(x) - \frac{1}{2}x + \frac{1}{2}\arctan(x) + C.$$

Comments: When you are using integration by parts, if one choice of u, dv does not seem to work, switch your expressions for u, dv. Also note carefully that if you let $u = x, dv = \arctan(x)dx$, then $v \neq \frac{1}{1+x^2}$ since that is the derivative of the $\arctan(x)$. Instead, to find v, you must integrate $\arctan(x)$.