M1220-2 **Quiz 3** **Spring 2005**

Quiz Scores (out of 10): n = 140; mean = 5.9
25th percentile = 3.5; median (50th percentile) = 6.0; 75th percentile = 8.5

1. (5 pts) Find $\frac{dy}{dx}$ in each of the following:

i. (2 pts) $y = (x^2+1)^{\sin x}$.

Using logarithmic differentiation we have:
$\ln y = \ln[(x^2+1)^{\sin x}] = (\sin x)\ln(x^2+1)$. Then differentiating implicitly with respect to x, we find: $\frac{1}{y}\frac{dy}{dx} = \sin x\left(\frac{2x}{x^2+1}\right) + (\cos x)\ln(x^2+1)$. Solving for $\frac{dy}{dx}$ and replacing y by $(x^2+1)^{\sin x}$ we have: $\frac{dy}{dx} = (x^2+1)^{\sin x}[\sin x\left(\frac{2x}{x^2+1}\right) + (\cos x)\ln(x^2+1)]$.

Comment: Logarithmic differentiation, and implicit differentiation are very useful tools!

ii. (3 pts) $y = \sin^{-1}(e^{2x})$

Using the Chain Rule we have:
$\frac{dy}{dx} = \frac{1}{\sqrt{1-(e^{2x})^2}}\frac{d(e^{2x})}{dx} = \frac{2e^{2x}}{\sqrt{1-(e^{4x})}}$

Comment: In differentiating $f(x) = e^{2x}$ we need the Chain Rule. $\frac{d(e^{2x})}{dx} = 2e^{2x}$. Also note that you need to memorize the derivatives of
$y = \sin^{-1}(x), y = \cos^{-1}(x)$, and $y = \tan^{-1}(x)$.

2. (3 pts) Evaluate the following definite integral.

$$\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$$

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If we let $u = \cos x$, then $du = -\sin x dx$. Note that $u = 1$, when $x = 0$, and $u =$, when $x = \pi/2$. The above integral becomes:

$$\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx = -\int_1^0 \frac{1}{1+u^2 du} = -\arctan u|_1^0 = -\arctan(0) + \arctan(1)$$

. Since $\arctan(0) = 0, \arctan(1) = \pi/4$, the above intergral equals $\pi/4$.

Comment: If you let $u = 1 + \cos^2 x$, then $du = 2\cos x \sin x dx$.

3. (2 pts) Suppose that $\frac{dy}{dx} + 2y = 3$ and that $y = 1$ when $x = 0$. Find y as a function of x.

The above equation is of the form, $\frac{dy}{dx} + \mathrm{P}(x)y = \mathrm{Q}(x)$ with $P(x) = 2$. Since $\int \mathrm{P}(x)dx = \int 2dx = 2x$, the integrating factor is e^{2x}. Multiplying the differential equation on both sides by this integrating factor we get:
$\frac{dy}{dx}e^{2x} + 2ye^{2x} = 3e^{2x}$, or $\frac{d(ye^{2x})}{dx} = 3e^{2x}$. Integrating both sides of the equation we have: $ye^{2x} = \int 3e^{2x}dx = (3/2)e^{2x} + \mathrm{C}, \mathrm{C}$ a constant. If we replace y by 1, and x by 0, then we get C = -1/2 and solving for y we have $ye^{2x} = (3/2) - (1/2)e^{-2x}$.

Comment: Note that $\int 3e^{2x}dx = \frac{3}{2}e^{2x}$. Remember that you can check your answer quickly in a small integral like this by simply differentiating. If you had $\int 3e^{2x}dx = 6e^{2x}$, then differentiating, you'd get $\frac{d(6e^{2x})}{dx} = 6[2e^{2x}] = 12e^{2x}$ and you know that you were off by a constant.