M1220-2 **Quiz 2** **Spring 2005**

Quiz Scores (out of 10): n = 141; Mean = 5.7; median = 6.0

1. (3 pts) Let $y = (\sin x)^{(x^2)}$. Find the derivative, $\frac{dy}{dx}$.

Some comments: Notice that here we have $y = f(x)^{g(x)}$, like the example we did in class where $y = x^x$. So we cannot use the differentiation rules for either the situation of $y = [f(x)]^a$ where a is a real number as in $y = (\sin x)^{3/4}$, or the situation of $y = a^{f(x)}$ where a is a real number as in $y = 5^{\sin x}$. Instead we use logarithmic differentiation:

1. Take the ln of both sides: $\ln y = \ln(\sin x)^{(x^2)} = x^2 \ln(\sin x)$.
2. Now differentiate both sides with respect to x:

$\frac{1}{y}\frac{dy}{dx} = x^2 \frac{d(\ln(\sin x))}{dx} + \ln(\sin x)\frac{d(x^2)}{dx}$.

Solving for $\frac{dy}{dx}$ we have:

$\frac{dy}{dx} = y[x^2 \frac{\cos x}{\sin x} + 2x\ln(\sin x)] = (\sin x)^{(x^2)}[x^2 \frac{\cos x}{\sin x} + 2x\ln(\sin x)]$.

2. (3 pts) Evaluate the following indefinite integral. Give your answer in terms of x.

$$\int \frac{e^{2/x}}{x^2} dx$$

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Let $u = 2/x$ so that $du = (-2/x^{-2})dx$. Then

$$\int \frac{e^{2/x}}{x^2} dx = -1/2 \int e^u du = -1/2(e^u) + C = -1/2(e^{2/x}) + C$$

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3.(2 pts) Assume the rate of change in the size of a population of animals is proportional to the size of the population. When initially observed 5 years ago, the population was 450. Now the population is 700.

What is the doubling time of the population? Leave your answer in terms of the ln of numbers specific to the problem.

Under the above assumption, the population size at time t is $y = y_0 e^{kt}$. We have $y_0 = 450$, and at time $t = 5, y = 700$. So, $700 = 450e^{5k}$, or $700/450 = e^{5k}$. Taking the ln of both sides of this equation we have: $\ln[700/450] = 5k$. Then $k = \frac{\ln[700/450]}{5}$.

Then, to find the doubling time, we can solve the equation $2 = 1e^{kt}$, or the equation, $900 = 450e^{kt}$ for t. Taking the ln of both sides of either equation, we find that $t = \frac{\ln 2}{k} = \frac{\ln 2}{\frac{\ln[700/450]}{5}} = \frac{5\ln 2}{\ln[700/450]}$.

4. (2 pt) Evaluate the following limit.

$$\lim_{n \to +\infty} \left(\frac{n+5}{n} \right)^{2n}$$

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Note that $\lim_{n \to +\infty} \left(\frac{n+5}{n} \right)^{2n} = \lim_{n \to +\infty} \left(1 + \frac{1}{n/5} \right)^{2n}$. But $n \to +\infty$ if and only if $n/5 \to +\infty$. Also, we can write $2n$ as $(n/5)10$. So we have,

$\lim_{(n/5) \to +\infty} \left(1 + \frac{1}{n/5} \right)^{(n/5)10} = [\lim_{(n/5) \to +\infty} \left(1 + \frac{1}{n/5} \right)^{(n/5)}]^{10} = e^{10}$.