M1220-2 **Quiz 1 Solutions** **Spring 2005**

Quiz Scores (out of 10): n = 144; Mean = 7.13; median = 7.5

1. (2 pts) Evaluate the following: $e^{4\ln 2}$ $\ln(e^{\cos\pi})$

Using the fact that $y = e^x, y = \ln x$ are inverse functions we have:
$e^{4\ln 2} = e^{\ln(2^4)} = (2^4) = 16; \ln(e^{\cos\pi}) = \cos\pi = -1$

2. (3 pts)Let $y = x^2 e^{\sin x}$. Find the derivative, $\frac{dy}{dx}$, and then evaluate the derivative when $x = \pi/2$.

Using the product rule and the fact that $\frac{de^u}{dx} = e^u \frac{du}{dx}$ we have:
$\frac{dy}{dx} = x^2 e^{\sin x}\cos x + 2xe^{\sin x}$. The value of this derivative at x = $\pi/2$ is:
$(\pi/2)^2 e^{\sin(\pi/2)}\cos(\pi/2) + 2(\pi/2)e^{\sin(\pi/2)} = 0 + (\pi)(e) = \pi(e)$.

Using logarithmic differentiation we have first that $\ln y = \ln[x^2 e^{\sin x}] = \ln x^2 + \ln e^{\sin x} = 2\ln x + \sin x$. Now, differentiating with respect to x, we have: $\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \cos x$. Solving for $\frac{dy}{dx}$ we have $\frac{dy}{dx} = [\frac{2}{x} + \cos x]y = x^2 e^{\sin x}\cos x + 2xe^{\sin x}$.

3. (3 pts) Evaluate the definite integral:

$$\int_0^1 \frac{x^3}{2x^4 + 1} dx$$

Let $u = 2x^4 + 1, du = 8x^3 dx$. When $x = 0, u = 1; when x = 1, u = 3$. So the above integral becomes:

$$\frac{1}{8}\int_1^3 \frac{du}{u} dx = \frac{1}{8}[\ln u|_1^3] = \ln 3/8$$

4.(2 pts) For all real x, let

$$h(x) = \int_0^x \ln(t^2 + 3)dt$$

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i. What is $h'(x)$?

By the First Fundamental Theorem of Calculus, $h'(x) = \ln(x^2 + 3)$. (Note that $f(t) = \ln(t^2 + 3)$ is a continuous function for all t so the theorem does apply.)

ii. The function $h(x)$ has an inverse function. Briefly state why. (Hint: Use $h'(x)$).

Since $h'(x) = \ln(x^2 + 3) > 0$ for all x, the function h is strictly increasing and by Theorem A (7.2), h has an inverse.