MATH 1260 - Test # 3 Sample Questions

Note: For full credit, explain your work and justify your answers.

(1) Calculate the double integral $\iint_{\mathcal{D}} 2xy\,dA$ where $\mathcal{D}$ is the triangle with vertices $(0,0)$, $(1,1)$, and $(2,0)$.

(2) Use a double integral to find the volume of the region beneath the plane with equation $y+z=1$ and above the triangle with vertices $(0,0)$, $(1,0)$, and $(0,1)$.

(3) Calculate the double integral $\iint_R y^2\,dA$ where R is the triangle with vertices $(2,0)$, $(0,1)$, and $(1,1)$.

(4) Consider two functions $f\colon \mathbb{R} \to \mathbb{R}$ and $g\colon \mathbb{R} \to \mathbb{R}$ of one variable. Show that

$$\int_0^1 \int_0^x (f''(x) - g''(y))\,dy\,dx = f(0) + g(0) + f'(1) + g'(0) - f(1) - g(1).$$

(5) Set up the triple integral $\iiint_{\mathcal{K}} f(x,y,z)\,dV$, where $\mathcal{K}$ is the region delimited by the planes $y=0$, $y=x$, $z=0$ and $x+y+2z=2$.

(6) Set up the triple integral $\iiint_{\mathcal{K}} f(x,y,z)dV$, where $\mathcal{K}$ is the solid tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(0,3,0)$, and $(0,0,6)$.

(7) Set up the triple integral $\iiint_K f(x,y,z)\,dV$ where K is the region below the graph of the function $4-x^2-y^2$ and above the x,y-plane.

(8) Calculate the work done to move an object around the circle C of radius 2 centered at $(0,0)$ in the vector field $\vec{F} =< \cos(x), x^2+\sin(y) >$.

(9) Calculate the integral $\int_C (x^2+y^2)\,ds$ where C is given by the two line segments from $(1,1)$ to $(-1,2)$ and from $(-1,2)$ to $(2,2)$.

(10) Calculate the flow of the vector field $\vec{F} =< xy, x^2 >$ through the line from $(0,1)$ to $(2,0)$.

(11) Calculate the integral $\int_C \vec{F}\cdot\vec{T}\,ds$ where $\vec{F} =< y^2+y, 2xy+z, y >$ and C is the triangle with vertices $(0,2,3)$, $(0,-1,1)$, and $(1,1,0)$.

(12) Calculate the integral $\int_C (x-y)\,ds$ where C is the circle C of radius 4 centered at $(1,0)$.

(13) Calculate the integral $\int_C \vec{F}\cdot\vec{N}\,ds$ where $\vec{F} =< x^2+y^2, x^2-y^2 >$ and C is the line segment from $(1,-1)$ to $(2,3)$.

(14) Calculate the work done to move an object around the triangle with vertices $(0,0)$, $(1,0)$, and $(0,2)$ when $\vec{F} =< x+y^2, 2xy+3x >$.

(15) Calculate the integral $\int_C 5x\,dx + 6xy^2\,dy + 2yz\,dy + y^2\,dz$ where C is the line segment from $(1,-1,2)$ to $(0,2,1)$.

(16) Calculate the integral $\int_C (x+y+z)\,ds$ where C is the circle in the x,y-plane of radius 3 with center $(0,2,0)$.

(17) Let C be the part of the graph of $x = y^3$ with $0 \leq x \leq 8$ followed by the line segment from $(8,2)$ to $(0,0)$. Calculate the flow of $\vec{F} =< 3xy, 2xy^2 >$ through C.

(18) Calculate the integral $\int_C (x+y)\,ds$, where C is the line segment from $(0,2)$ to $(-1,1)$.

(19) Calculate the integral $\int_C 2xy\,dx + (x^2 - y)\,dy$, where C is the line segment from $(1,1)$ to $(0,3)$.

(20) Calculate the work done in the vector field $\vec{F} =< 2xy^2, 2x^2y + z, y + x^2 >$ to move an object around the square with vertices $(0,1,0)$, $(1,1,0)$, $(1,1,1)$, and $(0,1,1)$.

(21) Calculate the flow of the vector field $\vec{F} =< x+y, x+y >$ through the curve C given by the portion of the graph of $y = x^2$ between $(0,0)$ and $(2,4)$.

(22) Calculate the integral

$$\int_C 5x\,dx + 6x^2z\,dx + y^2\,dy + 2yz\,dy + y^2\,dz + \sin(x^2y)\,dz + 2x^3\,dz,$$

where C is the circle of center $(0,0,2)$ and radius 15 in the plane $z = 2$.

(23) Prove that $\text{div}(\text{curl}(\vec{F})) = 0$ for every vector field $\vec{F}$.

(24) Prove that $\text{curl}(\text{grad}(f)) = \vec{0}$ for every function f.

(25) Calculate the integral $\int_C 12x\,dx + xy^2\,dy$, where C is the circle with center $(0,0)$ and radius 2.

(26) Let C be the circle of radius 2 and center $(0,1,0)$ in the y,z-plane. Show that

$$\int_C 4xy\,dx + e^{xyz}\,dx - y^2\,dx - 2x^2y\,dy + y\,dz - x\,dz = \int_C 2xy^2\,dx + z\,dx + 2xy\,dy - 2x^2\,dy - z\,dy.$$

(27) Let $f(x,y,z)$ be a function. Calculate $\text{div}(\text{grad}(f))$ in terms of f and its partial derivatives.

(28) Show that, if C is a closed curve in the xy-plane, then the flow of $\vec{F}$ through C equals

$$\iint_D \text{div}\vec{F}\,dA,$$

where D is the region enclosed by C.

Formula Sheet

- **Work**

$$W = \int_C \vec{F} \cdot \vec{T}\, ds = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt = \int_C P\, dx + Q\, dy,$$

where $\vec{F} =< P, Q >$ is a vector field.

- **Work in 3-Space**

$$W = \int_C \vec{F} \cdot \vec{T}\, ds = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt = \int_C P\, dx + Q\, dy + R\, dz,$$

where $\vec{F} =< P, Q, R >$ is a vector field.

- **Flow**

$$\int_C \vec{F} \cdot \vec{N}\, ds = \int_C -Q\, dx + P\, dy,$$

where $\vec{F} =< P, Q >$ is a vector field.

- **Fundamental Theorem of Line Integrals** If a vector field $< L, M >$ is the **gradient** of a function f (which means, $L = f_x$ and $M = f_y$), then

$$\int_C L\, dx + M\, dy = f(B) - f(A),$$

where A is the initial point of C and B is the final point of C.

- **Fundamental Theorem of Line Integrals in 3-Space** If a vector field $< L, M, N >$ is the **gradient** of a function f (which means, $L = f_x$, $M = f_y$, and $N = f_z$), then

$$\int_C L\, dx + M\, dy + N\, dz = f(B) - f(A),$$

where A is the initial point of C and B is the final point of C.

- **Green's Theorem** If C is a closed curve (which means, the intial point A is the same as the final point B of the curve C), then

$$\int_C L\, dx + M\, dy = \iint_R (M_x - L_y)\, dA,$$

where R is the region inside the closed curve C.

- **Divergence** If $\vec{F} =< P, Q, R >$ is a vector field, then

$$\text{div}(\vec{F}) = \nabla \cdot \vec{F} = P_x + Q_y + R_z.$$

- **Curl** If $\vec{F} =< P, Q, R >$ is a vector field, then

$$\text{curl}(\vec{F}) = \nabla \times \vec{F} =< R_y - Q_z, P_z - R_x, Q_x - P_y > .$$