MATH 1260 - Test # 1 Sample Questions

Formula Sheet

- The **dot product** of two plane vectors $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ is

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2.$$

- The **dot product** of two space vectors $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ is

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3.$$

- The **cross product** of two vectors $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ is

$$\vec{u} \cdot \vec{v} = \langle u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1 \rangle.$$

- The **projection** of a vector $\vec{u}$ onto a vector $\vec{v}$ is

$$\frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2} \vec{v}.$$

- The **determinant** of a 2 by 2 matrix is

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$$

- The **determinant** of a 3 by 3 matrix is

$$\det \begin{pmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \end{pmatrix} = \vec{a} \cdot (\vec{b} \times \vec{c}).$$

- Given a position vector $\vec{r}(t)$,
  - the **velocity vector** is $\vec{v}(t) = \vec{r}\,'(t)$,
  - the **speed** is $||\vec{v}(t)|| = \sqrt{\vec{v}(t) \cdot \vec{v}(t)}$,
  - the **acceleration** is $\vec{a}(t) = \vec{r}\,''(t) = \vec{v}\,'(t)$,
  - the **unit tangent vector** is $\vec{T}(t) = \dfrac{\vec{v}(t)}{||\vec{v}(t)||}$,
  - the **unit normal vector** is $\vec{N}(t) = \dfrac{(\vec{v}(t) \cdot \vec{v}(t))\vec{a}(t) - (\vec{v}(t) \cdot \vec{a}(t))\vec{v}(t)}{||(\vec{v}(t) \cdot \vec{v}(t))\vec{a}(t) - (\vec{v}(t) \cdot \vec{a}(t))\vec{v}(t)||}$,
  - the **unit binormal vector** is $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$, and
  - the **curvature** is $k(t) = \dfrac{||\vec{v}(t) \times \vec{a}(t)||}{||\vec{v}(t)||^3}$.

(1) Let $\vec{a} = \langle 2, -1, 3 \rangle$ and $\vec{b} = \langle 1, 0, -1 \rangle$. Find: (a) $\vec{a} \cdot \vec{b}$; (b) $\vec{a} \times \vec{b}$; (c) the projection of $\vec{a}$ in the direction of $\vec{b}$.

(2) (a) Find the line between the points $(1, 2, -1)$ and $(-3, 1, 0)$.
(b) Find the plane through the points $(1, 2, -1)$, $(-3, 1, 0)$, and $(0, -1, 2)$.

(3) An object is moving in the space with velocity at time t given by $\vec{v}(t) = \langle 2t^2 - 4t, \sin(t), 2t^3 - 3t^2 - 5 \rangle$. Knowing that the object is at the origin at time $t = 0$, find the position vector $\vec{r}(t)$ and the acceleration vector $\vec{a}(t)$.

(4) Sam is trying to throw a ball and hit a 5 m target which is 20 m away (horizontally). He throws the ball with a speed of 20 m/s and at an angle of 60^o from the horizontal, does he hit the target? (Acceleration caused by gravity is -9.8 m/s^2, and $\sin(60^o) = \sqrt{3}/2$, $\cos(60^o) = 1/2$)

(5) Write down equations for two planes so that the intersection of the two planes is a line with equation $\vec{r}(t) = \langle -4, 0, 3 \rangle + t\langle 1, 2, 3 \rangle$.

(6) Find the equation of the plane that containing the lines $\vec{r}(t) = \langle -3, 1, -5 \rangle + t\langle 2, -1, 2 \rangle$ and $\dfrac{x-3}{2} = y + 2 = \dfrac{z-1}{4}$.

(7) Let A be a 3 by 3 matrix, and let $\vec{a}$, $\vec{b}$, be the first two rows of A (each is a vector with 3 entries). Show that, if $\vec{b}$ is a multiple of $\vec{a}$, then the determinant of A is zero.

(8) Describe the cross section with respect to each coordinate plane of $z = (x + y)^2$.

(9) (a) Find the projection of $\langle 1, -2, 3 \rangle$ onto the y-axis. (b) Find a vector perpendicular to $\langle 1, -2, 3 \rangle$. (c) Find a vector perpendicular to both $\langle 1, -2, 3 \rangle$ and the vector you found in part (b).

(10) (a) Find the point where the two lines $\vec{r}(t) = (7 - 3t, 2t, 5 - 4t)$ and $\vec{r}(t) = (t, 2 + 2t, -1 - 2t)$ intersect. (b) Find the plane containing the two lines above.

(11) Find the intersection of the line $\vec{r}(t) = \langle 2t, 3 - t, t + 6 \rangle$ and the plane containing the point $(2, 1, 0)$ and the line $\langle t, -t, 1 + t \rangle$.

(12) Find the distance from the point $(1, 2, 3)$ and the plane $2x - y + z = 1$.

(13) Find a unit vector perpendicular to the vectors $\langle 1, 2, -3 \rangle$ and $\langle 5, 0, 1 \rangle$ that has a positive $\vec{k}$ coefficient.

(14) Determine whether the vectors $\langle 1, -1, -1 \rangle$, $\langle 0, 2, 7 \rangle$, and $\langle 1, 1, 6 \rangle$ are linearly dependent.

(15) If there exist two vectors, $\vec{u}$ and $\vec{v}$, whose lengths are equal, show that if they are not parallel, then $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are perpendicular.

(16) Find the area of the region formed by the part of the triangle with vertices $(0,0,0)$, $(4,2,4)$, $(0,2,6)$ which is not covered by the triangle with vertices $(0,0,0)$, $(2,1,2)$, $(0,1,3)$.

(17) Suppose that a particle follows a path $\langle \cos t, \sin t, t^3 \rangle$ and flies off at tangent at $t = \pi$. Where is the particle at $t = 2\pi$?

(18) A particle is at $\langle \sin t, \cos t, e^t \rangle$. Find its velocity, acceleration, and speed.

(19) A particle is at $\langle \cos t \sin t, t^3, e^{2t} \rangle$. Find velocity and acceleration vectors.

(20) Let $\vec{r}(t) = \langle \sin t, \cos t, t \rangle$. (a) Find: position, velocity, acceleration, and speed at $t = \pi$.
(b) Find the equation of the plane containing the velocity vector from (a) and the point $(3,1,2)$.
(c) What is the cross product of the velocity vector from (a) and the normal unit vector to the plane in (b) with positive $\vec{k}$?

(21) David is spinning a rock in his sling so that at any given time t (in seconds) the rock is at $\langle \sin(90t), -\cos(90t), \sin(90t) \rangle$. He lets the rock go after 10 seconds, and it strikes Goliath 3 seconds later. How far is David from Goliath? That is, how far did the rock fly?

(22) A person is running on a track in space with the position $x = 2t - 1$, $y = 36t^2 + 40$, $z = t$. What are the unit tangent vector, the unit normal vector, and the unit binormal vector at $t = 3$?

(23) Spell the professor's name.

(24) Given the equation $x = \cos t$, $y = \sin(2t)$, $z = 2$, find the curvature at time 3.

(25) Show that $\vec{B}$ is perpendicular to $\vec{T}$ and $\vec{N}$.

(26) Find the centripetal acceleration (towards the center of a circle) if $x = \cos(2t)$, $y = \sin(2t)$, and $z = t^2$ at $t = 3$.

(27) Calculate the determinant

$$\begin{vmatrix} x+1 & 1 & 1 \\ x & 0 & x-1 \\ 1 & 1 & x \end{vmatrix}.$$

(28) Find the volume of the parallelepiped spanned by the vectors $\vec{i} + \vec{j} + 2\vec{k}$, $2\vec{i} - \vec{j} + \vec{k}$, and $\vec{k} + \vec{i}$.

(29) Evaluate the determinant

$$\begin{vmatrix} x+2 & x-2 \\ x^4 & x^3 \end{vmatrix}.$$

(30) Show that, given three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, the determinant of the matrix with the three vectors as rows is zero if and only if the three vectors are on the same plane.