MATH 2210 - Final Exam - 5/5/2004

Note: Each problem is worth 10 points. For full credit, explain your work and justify your answers.

Name:

(1) Let $\vec{u} = \langle 1, -1, 2 \rangle$ and $\vec{v} = \langle 2, 0, 1 \rangle$.

(a) Find $\vec{u} \cdot \vec{v}$.

(b) Find $\vec{u} \times \vec{v}$.

(c) Find the projection of $\vec{u}$ onto $\vec{v}$.

(d) Find a vector perpendicular to both $\vec{u}$ and $\vec{v}$.

(2) Calculate the following limits:

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{-2xy}{x^2+y^2}$.

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^2+y^4}$.

(3) (a) Find the plane through the point $(0, 0, 0)$ and perpendicular to the line

$$\frac{x-2}{3} = y = \frac{z-1}{2}.$$

(b) Determine whether the plane above is perpendicular to the tangent plane to the function $f(x, y) = e^x + 3x^2y^2 - 2\ln(xy + 1) + \cos(y)$ at $(0, 0)$.

(4) Let $f(x, y, z) = (x^2 + yz, xyz + 3xz^2)$ and $g(x, y) = (y - x, xy)$. Find the derivative of the composition $g \circ f$ at the point $(0, 2, 3)$.

(5) (a) Find the stationary points of the function $f(x, y) = 2x^2 + x + y^2 - 2$ and classify them.

(b) Find the global maximum and minimum value of $f(x, y)$ on the region $x^2 + y^2 \leq 8$.

(6) Find the maximum volume of a rectangular box with no top and two dividers that can be built with 900 square feet of material.

(7) Calculate the double integral $\iint_R 2xy \, dA$ where R is the triangle with vertices $(2, 0)$, $(0, 2)$, and $(2, 2)$.

(8) Set up the triple integral $\iiint_K f(x,y,z)\,dV$ where K is the solid tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(0,1,0)$, and $(0,0,3)$.

(9) Find the flow of the vector field $\vec{F} = \langle 2xy - 2y, x^2 + \sin(x^3) + 3y^2 \rangle$ on the square with vertices $(0,2)$, $(1,2)$, $(1,3)$, $(0,3)$.

(10) Calculate

$$\int_C 2x\,dx + 3x^2\,dx + 2xy\,dx + x\,dy + x^2\,dy + x\,dz + 3z^2\,dz,$$

where C is the line segment from $(0,3,1)$ to $(-1,4,0)$ followed by the portion of the graph of $y = (x-1)^2$ in the x,y-plane with $-1 \leq x \leq 1$.

(Extra-credit) Verify by direct calculation that the gradient of $f(x, y) = y + 2x^2 - 2$ at $P = (1, 1)$ is perpendicular to the level curve at P. Sketch the gradient vector and the level curve.