MATH 2210 - Test 4 Sample Questions (First Part)

Calculate the following:

(1) The work done to move an object around the circle C of radius 2 centered at $(0,0)$ in the vector field $\vec{F} =< \cos(x), x^2 + \sin(y) >$.

(2) The integral $\int_C (x^2 + y^2)\, ds$ where C is given by the two line segments from $(1,1)$ to $(-1,2)$ and from $(-1,2)$ to $(2,2)$.

(3) The flow of the vector field $\vec{F} =< xy, x^2 >$ through the line from $(0,1)$ to $(2,0)$.

(4) The integral $\int_C \vec{F} \cdot \vec{T}\, ds$ where $\vec{F} =< y^2 + y, 2xy + z, y >$ and C is the triangle with vertices $(0,2,3)$, $(0,-1,1)$, and $(1,1,0)$.

(5) The integral $\int_C (x - y)\, ds$ where C is the circle C of radius 4 centered at $(1,0)$.

(6) The integral $\int_C \vec{F} \cdot \vec{N}\, ds$ where $\vec{F} =< x^2 + y^2, x^2 - y^2 >$ and C is the line segment from $(1,-1)$ to $(2,3)$.

(7) The work done to move an object around the triangle with vertices $(0,0)$, $(1,0)$, and $(0,2)$ when $\vec{F} =< x + y^2, 2xy + 3x >$.

(8) The integral $\int_C 5x\, dx + 6xy^2\, dy + 2yz\, dy + y^2\, dz$ where C is the line segment from $(1,-1,2)$ to $(0,2,1)$.

(9) The integral $\int_C (x + y + z)\, ds$ where C is the circle in the x, y-plane of radius 3 with center $(0,2,0)$.

(10) Let C be the part of the graph of $x = y^3$ with $0 \leq x \leq 8$ followed by the line segment from $(8,2)$ to $(0,0)$. Find the flow of $\vec{F} =< 3xy, 2xy^2 >$ through C.

FORMULAS

- **Line Integral**

$$\int_C f\,ds = \int_a^b f(\vec{r}(t))||\vec{r}\,'(t)||\,dt.$$

- **Work**

$$W = \int_C \vec{F}\cdot\vec{T}\,ds = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt = \int_C P\,dx + Q\,dy,$$

  where $\vec{F} =< P, Q >$ is a vector field.

- **Flow**

$$W = \int_C \vec{F}\cdot\vec{N}\,ds = \int_C -Q\,dx + P\,dy,$$

  where $\vec{F} =< P, Q >$ is a vector field.

- **Fundamental Theorem of Line Integrals** If a vector field $< L, M >$ is the **gradient** of a function f (which means, $L = f_x$ and $M = f_y$), then

$$\int_C L\,dx + M\,dy = f(B) - f(A),$$

  where A is the initial point of C and B is the final point of C.

- **Green's Theorem** If C is a closed curve (which means, the intial point A is the same as the final point B of the curve C), then

$$\int_C L\,dx + M\,dy = \int_R (M_x - L_y)\,dA,$$

  where R is the region inside the closed curve C.