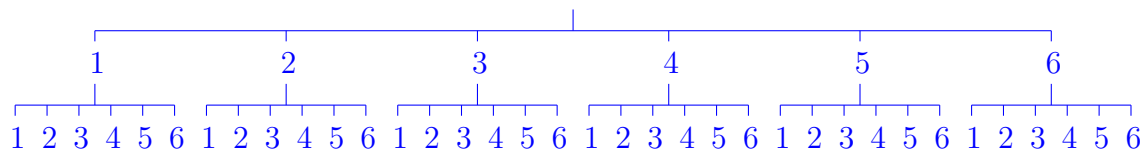


Solutions for Review Problems for Exam 2

Math 1040–1

1. You roll two fair dice.

(a) Draw a tree diagram for this experiment.



(b) What is the probability that you roll an even number on the first die and a 5 on the second die?

Three of the 36 possible outcomes meet this description, so the probability is $3/36 = 1/12 = 0.0833$.

(c) What is the probability that the numbers on the two dice add to 9? What is the probability that they add to 7?

Four of the outcomes add to 9, so the probability that the two numbers add to 9 is $4/36 = 1/9 = 0.111$. Six of the outcomes add to 7, so the probability that the two numbers add to 7 is $6/36 = 1/6 = 0.167$.

(d) What is the complement of the event that you roll an even number on the first die? What is the probability of this event?

The complement is “roll an odd number on the first die.” Its probability is $3/6 = 1/2 = 0.5$.

2. What is the probability that when drawing two cards from a standard deck (without replacement) you draw two aces? What is the probability that either the first card or the second card is an ace?

$$\begin{aligned} &P(\text{Ace on first draw and Ace on second draw}) \\ &= P(\text{Ace on first draw})P(\text{Ace on second draw} \mid \text{Ace on first draw}) \\ &= \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221} = 0.00452 \end{aligned}$$

$$\begin{aligned} &P(\text{Ace on first draw or Ace on second draw}) \\ &= P(\text{Ace on first draw}) + P(\text{Ace on second draw}) \\ &\quad - P(\text{Ace on first and second draws}) \\ &= \frac{4}{52} + \frac{4}{52} - \frac{1}{221} = 0.149 \end{aligned}$$

3. In a sales effectiveness seminar, a group of sales representatives tried two approaches to selling a customer a new automobile: the aggressive approach and the passive approach. For 1160 customers, the following record was kept:

	Sale	No Sale	Total
Aggressive	270	310	580
Passive	416	164	580
Total	686	474	1160

- (a) What is the probability that a sale was made to a randomly chosen customer?

Sales were made to 686 out of 1160 customers, so the probability is $\frac{686}{1160} = 0.591$.

- (b) What is the probability that a sale was made to a randomly chosen customer, knowing that a sales representative tried the aggressive approach?

We know that the customer was one of the 580 for which the aggressive approach was used. Sales were made to 270 of these customers, so the probability is $\frac{270}{580} = 0.466$.

- (c) Are the events “sale” and “aggressive approach” independent or dependent? Explain.

The events are dependent, since $P(\text{Sale}) \neq P(\text{Sale} | \text{Aggressive})$ from parts 3a and 3b.

- (d) What is the probability that an aggressive approach was used and a sale was made for a customer chosen at random?

270 out of 1160 customers meet this description, so the probability is $\frac{270}{1160} = 0.233$.

- (e) What is the probability that an aggressive approach was used or a sale was made for a customer chosen at random?

$$\begin{aligned} P(\text{Aggressive or Sale}) &= P(\text{Aggressive}) + P(\text{Sale}) - P(\text{Aggressive and Sale}) \\ &= \frac{580}{1160} + \frac{686}{1160} - \frac{270}{1160} = 0.859 \end{aligned}$$

4. There are three nursing positions to be filled at a certain hospital: a day nursing supervisor, a night nursing supervisor, and a nursing coordinator position. There are 15 applicants qualified for all three positions. Determine the number of different ways the positions can be filled by these applicants.

Since the position in which a person ends up matters (instead of just whether they are selected for a job or not), we are looking for ${}_{15}P_3 = \frac{15!}{(15-3)!} = 2730$.

5. One professor grades homework by randomly choosing 5 out of 12 homework problems to grade.
- (a) How many different groups of 5 problems can be chosen from the 12 problems?

Since we are looking at the number of ways questions can be chosen, order does not matter, so we need ${}_{12}C_5 = \frac{12!}{(12-5)!5!} = 792$.

- (b) Sylvia only completed 7 problems from the assignment. What is the probability that she did not miss any questions that will be graded?

By finding the number of ways that 5 questions can be chosen from the 7 that Sylvia completed, we will find the number of ways that Sylvia will not miss questions. There are ${}_{7}C_5 = \frac{7!}{(7-5)!5!} = 21$ ways. To find the probability, we need to divide this number by the total number of ways we found in part 5a, so we get $\frac{21}{792} = 0.0265$.

6. Suppose the parole board is examining five prisoners up for parole. Let x represent the number of prisoners out of five on parole who become repeat offenders while on parole. Then, the probability distribution for x is

x	0	1	2	3	4	5
$P(x)$	0.237	0.395	0.264	0.088	0.015	0.001

- (a) What is the probability that at least 3 prisoners will become repeat offenders?

$$P(x \geq 3) = P(3) + P(4) + P(5) = 0.088 + 0.015 + 0.001 = 0.104$$

- (b) What is the average number of repeat offenders?

$$\begin{aligned} \mu &= \sum xP(x) \\ &= 0 \cdot 0.237 + 1 \cdot 0.395 + 2 \cdot 0.264 + 3 \cdot 0.088 + 4 \cdot 0.015 + 5 \cdot 0.001 \\ &= 1.252 \end{aligned}$$

- (c) What is the standard deviation for the number of repeat offenders?

We will first calculate the variance, then take the square root:

$$\begin{aligned} \sigma^2 &= \sum (x - \mu)^2 P(x) \\ &= (0 - 1.252)^2 \cdot 0.237 + (1 - 1.252)^2 \cdot 0.395 + (2 - 1.252)^2 \cdot 0.264 \\ &\quad + (3 - 1.252)^2 \cdot 0.088 + (4 - 1.252)^2 \cdot 0.015 + (5 - 1.252)^2 \cdot 0.001 \\ &= 0.940 \\ \sigma &= \sqrt{0.940} = 0.970 \end{aligned}$$

7. The Orchard Café has found that about 5% of the diners who make reservations do not show up. If 82 reservations have been made, what is the probability that exactly 4 diners with reservations do not show up?

The number of diners with reservations who do not show follows a binomial distribution with $n = 82$, $p = 0.05$, and $q = 0.95$.

The probability that exactly 4 miss their reservations is $P(4) = \frac{82!}{(82-4)!4!} 0.05^4 \cdot 0.95^{78} = 0.200$.

8. On the leeward side of the island of Oahu, in the small village of Nanakuli, about 80% of the residents are of Hawaiian ancestry (Source: *The Honolulu Advertiser*). Let x represent the number of people you meet until you encounter the first person of Hawaiian ancestry in the village of Nanakuli. What is the probability that the first person you encounter with Hawaiian ancestry is at least the fourth that you meet?

This describes a geometric distribution, since we are asked for the “first time” that an event happens. The probability of success is $p = 0.8$ and $q = 0.2$. We want to find

$$\begin{aligned} P(x \geq 4) &= 1 - P(x \leq 3) = 1 - P(1) - P(2) - P(3) \\ &= 1 - 0.8 \cdot 0.2^0 - 0.8 \cdot 0.2^1 - 0.8 \cdot 0.2^2 \\ &= 0.008 \end{aligned}$$

9. *USA Today* reported that for all airlines, the number of lost bags was 12.78 per 1000 passengers in December. What is the probability that out of 1000 passengers, no bags are lost? What is the probability that exactly 13 bags are lost?

If we let x represent the number of bags lost, it follows a Poisson distribution with $\mu = 12.78$. Then,

$$\begin{aligned} P(0) &= \frac{12.78^0 e^{-12.78}}{0!} = 0.00000282 \text{ and} \\ P(13) &= \frac{12.78^{13} e^{-12.78}}{13!} = 0.110 \end{aligned}$$