

Solutions to Review Problems for Sections 4.2 and 4.3

Math 1040–1

1. At Fontaine Lake Camp on Lake Athabasca in northern Canada, history shows that about 30% of the guests catch lake trout over 20 pounds on a 4-day fishing trip. Let x be a random variable that represents the first trip to Fontaine Lake Camp on which a guest catches a lake trout over 20 pounds. Find the probability that a guest catches a lake trout weighing at least 20 pounds for the first time on trip number 3. What is the probability that it takes more than three trips for a guest to catch a lake trout weighing at least 20 pounds?

The random variable x satisfies the requirements for a Geometric distribution since we are looking at the “first time” an event happens.

The probability of “success” on one trip is $p = 0.3$, and $q = 1 - p = 0.7$. The probability that a guest catches a lake trout weighing at least 20 pounds for the first time on trip number 3 is $P(3) = 0.3 \cdot 0.7^{3-1} = .147$.

The probability that it takes more than three trips for a guest to catch a lake trout weighing at least 20 pounds is $P(x > 3) = 1 - P(x \leq 3) = 1 - P(1) - P(2) - P(3) = 1 - 0.3 \cdot 0.7^{1-1} - 0.3 \cdot 0.7^{2-1} - 0.3 \cdot 0.7^{3-1} = 0.343$.

2. The *Denver Post* reported that, on average, a large shopping center had an incident of shoplifting caught by security once every three hours. Let x be the number of shoplifting incidents caught by security in one day during which the center is open from 10 a.m. to 9 p.m. What is the probability that at least one shoplifting incident will be caught by security? What is the probability that at least three shoplifting incidents will be caught?

The random variable x satisfies the requirements for a Poisson distribution, since we are counting the number of “events” (here, shoplifting incidents) in an interval (a day). In order to calculate probabilities using our formula, we will need to know the average number of shoplifting incidents in one day (the 11 hours between 10 a.m. and 9 p.m.). To get this, we use ratios to get that $\mu = \text{average per hour} \cdot \text{number of hours} = \frac{1}{3} \cdot 11 = \frac{11}{3} = 3.66667$.

The probability that at least one shoplifting incident will be caught by security is most easily calculated by using the probability of the complement of this event. $P(x \geq 1) = 1 - P(0) = 1 - \frac{3.66667^0 e^{-3.66667}}{0!} = 1 - 0.0256 = 0.9744$.

The probability that at least three shoplifting incidents will be caught is also more easily calculated by using the complement. $P(x \geq 3) = 1 - P(x \leq 2) = 1 - P(0) - P(1) - P(2) = 1 - \frac{3.66667^0 e^{-3.66667}}{0!} - \frac{3.66667^1 e^{-3.66667}}{1!} - \frac{3.66667^2 e^{-3.66667}}{2!} = 0.7089$.

3. The quality control inspector of a production plant will reject a batch of syringes if two or more defective syringes are found in a random sample of eight syringes taken from the batch. Suppose that the batch contains 1% defective syringes. What is the probability that the batch will be accepted?

By letting x represent the number of defective syringes in the random sample, x is a binomial random variable with $p = 0.01$, $q = 0.99$, and $n = 8$.

We want to find $P(x \leq 1) = P(0) + P(1) = \frac{8!}{(8-0)!0!} 0.01^0 \cdot 0.99^8 + \frac{8!}{(8-1)!1!} 0.01^1 \cdot 0.99^7 = 0.9973$.

4. *USA Today* reported that the U.S. (annual) birth rate is about 16 per 1000 people, and the death rate is about 8 per 1000 people. In a community of 1500 people, what is the probability of there being 16 births in a given year? What is the probability of 10 deaths?

If we let x represent the number of births and y represent the number of deaths in this community, both are Poisson random variables since we are counting the number of events (births or deaths) in a given interval (a year). The mean of x is $\mu_x = \frac{16}{1000} \cdot 1500 = 24$ and the mean of y is $\mu_y = \frac{8}{1000} \cdot 1500 = 12$.

The probability of 16 births in a year is $P(16) = \frac{24^{16}e^{-24}}{16!} = 0.02186$.

The probability of 10 deaths in a year is $P(10) = \frac{12^{10}e^{-12}}{10!} = 0.1048$.