Quiz 5 Math 1040–1 July 13, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. Unless otherwise directed, give each decimal approximation rounded to at least three decimal places. Each part of each question is worth 10 points.

1. Calculate the probability that a standard normal random variable is between z = -1.96 and z = 1.96. (#35 from 5.1 with a variation on wording)

We look up the z-scores -1.96 and 1.96 on the table and get the probabilities 0.0250 and 0.9750. Since we want the probability of being between the two z-scores, we subtract the probabilities to get 0.9750 - 0.0250 = 0.9500.

- 2. A survey was conducted to measure the heights of U.S. men. In the 20–29 age group, the heights were normally distributed, with a mean of 69.9 inches and a standard deviation of 3 inches. A study participant is randomly selected. (#13, parts a and b from 5.2 combined with #32 from 5.3)
 - (a) Find the probability that his height is less than 66 inches.

First, we find the z-score, $z = \frac{66-69.9}{3} = -1.3$, then use this look up the probability in the table and see that the answer is 0.0968.

(b) Find the probability that his height is between 66 and 72 inches.

We already calculated the z-score for 66, z = -1.3. The z-score for 72 is $z = \frac{72-69.9}{3} = 0.7$. Looking these up on the table, we get the probabilities 0.7580 and 0.0968. Since we are asked about being between two heights, we subtract those probabilities to get 0.7580 - 0.0968 = 0.6612.

(c) What height represents the 90th percentile?

The probability closest to 0.90 on the table is 0.8997, which happens at z = 1.28. Solving the equation $1.28 = \frac{x-69.9}{3}$ for x, we get that the 90th percentile is 73.74.

(d) What height represents the first quartile?

The probability closest to 0.25 on the table is 0.2514, which happens at z = -.67. Solving the equation $-.67 = \frac{x-69.9}{3}$, we get that Q_1 is 67.89.

3. The heights of women in the United States (ages 20–29) have a mean of 64.3 inches and a standard deviation of 2.6 inches. A random sample of 60 women in this age group is selected. What is the probability that the mean height for the sample is greater than 66 inches? (#29 from 5.4)

Since we are asked about the mean (not an individual observation), we need to calculate the standard deviation for the mean: $\sigma_{\bar{x}} = 2.6/\sqrt{60} = 0.3357$. Now, we calculate the z-score, $z = \frac{66-64.3}{0.3357} = 5.06$. This number is larger than the z-scores on the table, so we can assume that the table would give us (approximately) 1. Since we are asked about "greater than," we need to subtract this probability from 1, so we get 1 - 1 = 0.