Quiz 4 Key
Math 1040–1
June 22, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. Unless otherwise directed, give each decimal approximation rounded to at least three decimal places.

Formulas:

${}_nP_r = \frac{n!}{(n-r)!}$

Ways to order n objects with n_1 alike, n_2 alike, $\ldots$, and n_k alike $= \frac{n!}{n_1!\,n_2!\cdots n_k!}$

${}_nC_r = \frac{n!}{(n-r)!\,r!}$

$\sigma^2 = \sum (x-\mu)^2 P(x)$

1. A company that makes cartons finds that the probability of producing a carton with a puncture is 0.05, the probability that a carton has a smashed corner is 0.08, and the probability that a carton has a puncture and has a smashed corner is 0.004.

   Are the events "selecting a carton with a puncture" and "selecting a carton with a smashed corner" mutually exclusive? Find the probability that a carton chosen at random has a puncture or a smashed corner. (#15 from 3.3)

   The events are not mutually exclusive, since it is possible for them to both happen at once.

$$\begin{aligned} P(\text{puncture or smashed corner}) &= P(\text{puncture}) + P(\text{smashed corner}) \\ &\quad - P(\text{puncture and smashed corner}) \\ &= 0.05 + 0.08 - 0.004 = 0.126 \end{aligned}$$

2. In a state lottery, you choose 5 different numbers out of 40. To win the top prize, your numbers must match the 5 numbers chosen by the lottery in any order. You purchase one lottery ticket. What is the probability that you will win the top prize? (#53 from 3.4)

   Since numbers can be picked in any order, there are ${}_{40}C_5 = \frac{40!}{35!5!} = 658008$ ways to choose 5 numbers at a time out of 40. Since there is only one winning combination, the probability of selecting the winning combination is $1/658008 = 0.00000152$.

3. Find the number of distinguishable ways that the letters in "statistics" can be arranged. (#27 from 3.4)

   Of the 10 letters in the word, there are 3 s's, 3 t's, 1 a, 2 i's, and 1 c, so the total number of distinct arrangements of the letters is $\frac{10!}{3!3!1!2!1!} = 50400$.

4. Let x represent the amount of snow (in inches) that fell in Nome, Alaska, last winter. Determine whether x is discrete or continuous. Explain your reasoning. (#19 from 4.1)

   x is continuous, since the amount of snow in a year can be any value in the interval from 0 to infinity. (Note: Saying that x can take an infinite number of values x is not sufficient, since the set of whole numbers is infinite, but is discrete.)

5. The number of televisions per household in a small town of 2600 households are: (#29 from 4.1)

Televisions	0	1	2	3
Households	26	442	728	1404

   (a) Let x represent the number of televisions for a randomly selected household. Construct a probability distribution for the above data:

x	0	1	2	3
$P(x)$	$\frac{26}{2600} = 0.01$	$\frac{442}{2600} = 0.17$	$\frac{728}{2600} = 0.28$	$\frac{1404}{2600} = 0.54$

   (b) Calculate the mean of the probability distribution.

$$\mu = \sum xP(x) = 0 \cdot 0.01 + 1 \cdot 0.17 + 2 \cdot 0.28 + 3 \cdot 0.54 = 2.35$$

   (c) Calculate the standard deviation of the probability distribution.

$$\begin{aligned}
\sigma^2 &= \sum (x-\mu)^2 P(x) \\
&= (0-2.35)^2 \cdot 0.01 + (1-2.35)^2 \cdot 0.17 + (2-2.35)^2 \cdot 0.28 \\
&\quad + (3-2.35)^2 \cdot 0.54 \\
&= 0.6275 \\
\sigma &= \sqrt{0.6275} = 0.792
\end{aligned}$$