Section 4.3, More Discrete Probability Distributions

1 Geometric Distribution

A **geometric distribution** is a discrete probability distribution of a random variable x that satisfies the following conditions:

- A trial is repeated until a success occurs.

- The repeated trials are independent of each other, and have the same probability of success, p.

- The random variable x represents the number of the trial in which the first success occurs.

Then, the probability that the first success will occur on trial x is $P(x) = pq^{x-1}$, with $q = 1 - p$ and $x \geq 1$. The mean of a geometric distribution is $\mu = \frac{1}{p}$, the variance is $\sigma^2 = \frac{1-p}{p^2}$, and the standard deviation is $\sigma = \frac{\sqrt{1-p}}{p}$.

Examples

1. Bitter pit is a disease of apples resulting in a soggy core, caused by either overwatering or a calcium deficiency in the soil. Approximately 3.6% of all (untreated) Jonathan apples had bitter pit in a study conducted by the botanists Ratkowsky and Martin (*Australian Journal of Agricultural Research*, Vol. 25, pp. 783–790). Let x represent the first Jonathan apple chosen at random that has bitter pit.

   (a) What is the probability that the first two apples chosen do not have bitter pit, and the third one does?

   Here, $p = 0.036$ and $q = .964$, so $P(3) = 0.036 \cdot 0.964^2 = 0.03345$.

   (b) What is the probability that the first three apples chosen do not have bitter pit?

   Here, we are not directly asked to wait until we have a success, but we can translate the problem to finding $P(x \geq 4) = 1 - P(x \leq 3) = 1 - P(1) - P(2) - P(3) = 1 - 0.036 - 0.036 \cdot 0.964 - 0.036 \cdot 0.964^2 = 0.8958$.

   (c) What is the average number of apples that you need to pick until you find one with bitter pit? (include the one with bitter pit)

   We are asked for the mean of x, so $\mu = \frac{1}{0.036} = 27.778$.

2. According to the National Conference on Bar Examiners, about 57% of all people who take the state bar exam pass. Bob is a recent law school graduate who intends to take the state bar exam.

   (a) How many times should Bob plan to take the exam?

   The average person will take the bar exam $\mu = \frac{1}{0.57} = 1.754$ times, so Bob should probably plan to take the exam two times.

   (b) What is the probability that Bob passes the exam either the first or second time that he takes it?

   We want $P(1 \text{ or } 2) = P(1) + P(2) = 0.57 + 0.57 \cdot 0.43 = 0.8151$.

2   Poisson Distribution

A **Poisson distribution** is a discrete probability distribution of a random variable x that satisfies:

- The experiment counts the number of times, x, that an event occurs in a given interval (the interval can measure time, area, volume, etc.).

- The probability of the event occurring is the same for each interval.

- The number of occurrences in one interval is independent of the number of occurrences in other intervals.

Then, the probability of x occurrences in an interval is $P(x) = \frac{\mu^x e^{-\mu}}{x!}$, where $e \approx 2.71828$ and μ is the average number of occurrences per interval unit. Note that $x \geq 0$ The mean of x is μ, the variance is $\sigma^2 = \mu$, and the standard deviation is $\sigma = \sqrt{\mu}$.

Examples

1. A certain store advertises that every one of their cashiers averages one minute per customer. What is the probability that you will be through the checkout line (and the person behind you in line will not be done) after 5 minutes if you are currently the fifth person in line?

   We aren't directly given μ. We can calculate it using that the "interval" is 5 minutes, and 5 customers will be able to pass through the line on average, so $\mu = 5$. Then, $P(5) = \frac{5^5 \cdot e^{-5}}{5!} = 0.17547$.

   Note: If we had wanted the probability that you were done, and it did not matter whether the person behind you was done or not, you would actually want to calculate $P(x \geq 5)$.

2. Insurance agents claim that a Denver, Colorado, can expect to replace his or her roof once every 10 years due to hail damage. What is the probability that in 12 years a homeowner in Denver will need to replace the roof twice because of hail?

   Since we are asked information about what happens in 12 years, but are given information about what happens in 10 years, we need to find the average number of times a roof will need to be replaced in 12 years. So, using ratios, we see that $\mu = 1 \cdot \frac{12}{10} = 1.2$. Now, we want $P(2) = \frac{1.2^2 e^{-1.2}}{2!} = 0.21686$. So, there is a 21.686% chance that a homeowner in Denver will need to replace their roof twice in 12 years.

Note: The book has a good table about the differences between Binomial, Geometric, and Poisson Distributions on page 221.