Examples for Chapter 5– Normal Probability Distributions
Math 1040–1

Section 5.1

1. Heights of males at a certain university are approximately normal with a mean of 70.9 inches and a standard deviation of 2.9 inches. Find the z-score for a male who is 6 feet tall.

2. Let z be a standard normal random variable:

   (a) Find the probability that z falls below 0.

   (b) Find the probability that z falls below 2.74.

   (c) Find the probability that z falls below -0.93.

   (d) Find the probability that z is at least 0.62.

   (e) Find $P(z \geq -2.6)$.

   (f) Find $P(-0.24 \leq z \leq 0.43)$.

   (g) Find $P(z = 1)$.

   (h) Find $P(z \leq -4)$.

   (i) Find x such that $P(z \leq x) = 0.9222$.

   (j) Find the first quartile for a standard normal distribution.

Sections 5.2 and 5.3

1. Find the 7th percentile for a standard normal distribution.

2. ACT scores are approximately normal with a mean of 21 and a standard deviation of 4.7.

   (a) What is the probability that a student chosen at random receives less than a 26 on the ACT?

   (b) What is the probability that a student chosen at random receives at least a 24 on the ACT?

   (c) What is the third quartile for the ACT?

   (d) What is the 80th percentile for the ACT?

3. A brand of cassette decks had a deck life that was normally distributed with a mean of 2.3 years and a standard deviation of 0.4 years.

   (a) What is the probability that the cassette deck will break down less than one year after purchase?

   (b) What is the probability that the cassette deck will last for at least 3 years after purchase?

   (c) What is the 99th percentile for the life of these cassette decks?

4. A pizza parlor franchise specifies that the amount of cheese on a large pizza should be approximately normal with a mean of 8 ounces and a standard deviation of 0.5 ounces.

   (a) What is the probability that a pizza chosen at random has less than 7.3 ounces of cheese?

   (b) What is the probability that a pizza has more than 8.95 ounces of cheese?

   (c) What is the probability that a pizza contains between 7.6 and 8.3 ounces of cheese?

   (d) What is the probability that a pizza has exactly 8 ounces of cheese?

   (e) What is the least amount of cheese that can be on a pizza that will still place in the top 10% of cheesiest pizzas?

   (f) Between what two values does the middle 70% of cheese lie?

Section 5.4

1. A measurement from a population has population mean 6 and standard deviation 2. What are the mean and standard error of $\bar{x}$ when $n = 4$? When $n = 100$? When $n = 400$?

2. For the population of farm workers in New Zealand, suppose that weekly income has a distribution that is skewed right with a mean of $\mu = \$500$ (N.Z. dollars) and a standard deviation of $\sigma = \$160$. A survey of 100 farm workers is taken, including information on their weekly income.

   (a) What are the mean and standard error of the sampling distribution of $\bar{x}$?

   (b) What is the probability that the mean weekly income of these 100 workers is less than 448?

   (c) What is the probability that the mean weekly income of these 100 workers is between 480 and 520?

3. The heights of 18-year-old men are approximately normally distributed with mean of 68 inches and standard deviation of 3 inches.

   (a) What is the probability that an 18-year-old man selected at random is between 67 and 69 inches tall?

   (b) For a sample of 36 18-year-old men, what is the probability that the average of their heights is between 67 and 69 inches?

4. For people under 50, the level of glucose in the blood (in milligrams per deciliter of blood) after a 12-hour fast have a standard deviation of 25 and a mean of μ. What is the probability that, for a sample of size 49 readings, the sample mean is within 7 of μ?

Section 5.5

1. The owner of a new apartment building must install 25 water heaters. A certain brand is guaranteed for 5 years, but the probability that it will last 10 years is 0.25. What is the approximate probability that 8 or more of the hot water heaters will last at least 10 years?

2. From many years of observation, a biologist knows that the probability is only 0.65 that any given Arctic tern will survive the migration from its summer nesting area to its winter feeding grounds. A random sample of 500 Arctic terns were banded at their summer nesting area. What is the approximate probability that between 310 and 340 of the banded Arctic terns will survive the migration?

3. A professor is giving an exam to a class of 200 students. From past semesters, he knows that 60% of students taking this course receive at least a 70% on this exam. What is the probability that at least 130 of his students will receive a 70% on the test?

4. You flip a weighted coin 10 times (it gives tails 30% of the time, and heads 70%). What is the probability that you will receive at least 9 heads?