

## Review Problems for Exam 1

Math 1100-4

Tuesday, February 7, 2012

### Examples

1. Calculate each of the following limits:

(a)  $\lim_{x \rightarrow 4} x^3 - 2x + 2$

(b)  $\lim_{x \rightarrow 5} \frac{x^2 - 7x + 10}{x^2 - 25}$

(c)  $\lim_{x \rightarrow -2} \frac{x^2 + x + 2}{x^2 - 4x - 12}$

(d)  $\lim_{x \rightarrow 6} \begin{cases} x - 6, & x \leq 6 \\ x^2 + 36, & x > 6 \end{cases}$

(e)  $\lim_{x \rightarrow 6^+} \begin{cases} x - 6, & x \leq 6 \\ x^2 + 36, & x > 6 \end{cases}$

(f)  $\lim_{x \rightarrow \infty} \frac{2x^3 - 5x^2 + x - 6}{3x^3 + 8x + 4}$

2. Find any horizontal asymptotes of  $f(x) = \frac{2x^3 - 5x^2 + x - 6}{3x^3 + 8x + 4}$ .

3. Determine whether the function  $f(x) = \frac{x^3 + 3x^2 - 4x}{x^2 - 16}$  is continuous. If it is not, identify the  $x$ -value(s) at which it is discontinuous. If it is continuous for all values of  $x$ , state the conditions needed for continuity.

4. What are the vertical asymptote(s) of the function  $y = \frac{x^3 + 3x^2 - 4x}{x^2 - 16}$ ?

5. Find the average rate of change of  $y = 2x^3 - 4x^2 + 1$  on the interval  $[-1, 2]$ .

6. Let  $f(x) = 2x^2 - 3x + 4$ .

(a) Use the limit definition of the derivative to calculate  $f'(x)$ .

(b) What is the slope of the line tangent to  $y = f(x)$  when  $x = 2$ ?

7. Calculate the derivative of each of the following functions:

(a)  $f(x) = \pi^4$

(b)  $y = 2x^3 - \frac{3}{x^5} + 3$

(c)  $y = \frac{5\sqrt{1-x^3}}{6}$

(d)  $g(t) = \frac{1-t}{(1+t)^2}$

- (e)  $h(z) = (z + 1)^2(z^2 - z + 2)^3$
8. What is the instantaneous rate of change of  $y = x^2 + \frac{2}{x}$  with respect to  $x$  when  $x = 3$ ?
9. Calculate the indicated higher-order derivative of each of the following functions:
- (a)  $f''(x)$  if  $f(x) = 2x^7 - \frac{4}{x^5}$
- (b)  $\frac{d^3y}{dx^3}$  if  $y = (2x - 6)^5$
10. Suppose that the total profit function for a commodity is  $P(x) = 22x - 0.001x^2 - 7$ , where  $P$  is the profit in dollars when  $x$  units are sold.
- (a) Find the marginal profit function.
- (b) Calculate the marginal profit when  $x = 10$ . What does this predict about the sale of the next unit?
- (c) Calculate  $P(11) - P(10)$ . What does this represent?
- (d) What is the rate of change of the marginal profit when 10 units are sold?
11. Let  $y = 2x^3 - 3x^2 - 1$
- (a) Find the  $y$ -intercept of the function.
- (b) Find the critical value(s) of the function.
- (c) Where is the function increasing? Where is it decreasing?
- (d) Find the relative maxima and minima of  $y$ . Be sure to justify your answer with either the First Derivative Test or the Second Derivative Test.
- (e) Where is the graph of  $y$  concave up? Where is it concave down? What are its point(s) of inflection?
- (f) Using the information found in the previous parts of this question, sketch the graph of  $y = 2x^3 - 3x^2 - 1$ .