Study Guide for Exam 3
Math 1100–4

Integration Formulas: Let C represent the constant of integration:

- $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$.
- $\displaystyle\int cu(x)\,dx = c\int u(x)\,dx$ if c is a constant.
- $\displaystyle\int \big(u(x) \pm v(x)\big)\,dx = \int u(x)\,dx \pm \int v(x)\,dx$ if $u(x)$ and $v(x)$ are integrable functions.
- $\displaystyle\int 1\,dx = x + C$.
- $\displaystyle\int 0\,dx = C$.
- $\displaystyle\int u'(x)\big[u(x)\big]^n\,dx = \frac{\big[u(x)\big]^{n+1}}{n+1} + C$ for $n \neq -1$.
- $\displaystyle\int u'(x)\,e^{u(x)}\,dx = e^{u(x)} + C$.
- $\displaystyle\int \frac{u'(x)}{u(x)}\,dx = \ln|u(x)| + C$.

Other Formulas to Know:

- $\displaystyle\sum_{i=1}^{n} 1 = n$.
- $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.
- Area under $f(x)$ from $x = a$ to $x = b$ with a right-hand approximation if $f(x) \geq 0$ on $[a, b]$: $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{b-a}{n}\right)f\left(a + i\frac{b-a}{n}\right)$.
- Area between $f(x)$ and $g(x)$ from $x = a$ to $x = b$: $\int_a^b (f(x) - g(x))\,dx$ if $f(x) \geq g(x)$ on $[a, b]$.
- Average value of $f(x)$ from $x = a$ to $x = b$: $\displaystyle\frac{1}{b-a}\int_a^b f(x)\,dx$.
- Total Income from $t = 0$ to $t = k$ for the rate of continuous income flow $f(t)$: $TI = \int_0^k f(t)\,dt$.
- Present Value from $t = 0$ to $t = k$ for the rate of continuous income flow $f(t)$ with an interest rate of r, compounded continuously: $PV = \int_0^k f(t)e^{-rt}\,dt$.

- Future Value from $t = 0$ to $t = k$ for the rate of continuous income flow $f(t)$ with an interest rate of r, compounded continuously: $FV = e^{rk} \int_0^k f(t)e^{-rt}\, dt = e^{rk} \cdot PV$.
- $\int_a^\infty f(x)\, dx = \lim_{b\to\infty} \int_a^b f(x)\, dx$.
- $\int_{-\infty}^b f(x)\, dx = \lim_{a\to-\infty} \int_a^b f(x)\, dx$.

Section 12.1

- Know the definition of the (indefinite) integral.
- Be able to integrate polynomials, radicals, and fractions if they can be written as the sum of different powers of x, where none of the exponents are -1. Remember to include the "$+C$."
- Given the marginal revenue function, be able to calculate the Revenue function, using that $R(0) = 0$. (Note: This does not mean that the constant of integration $C = 0$, in general.)

Section 12.2

- Be able to integrate functions of the form $u'(x)[u(x)]^n$ for $n \neq -1$, and recognize when to multiply and divide by a constant to make $u'(x)$ appear within the integral sign. Remember to include the "$+C$."

Section 12.3

- Be able to integrate exponential functions. Remember to include the "$+C$."
- Also be able to integrate functions of the form $\frac{u'(x)}{u(x)} = u'(x) \cdot [u(x)]^{-1}$. Remember to include the "$+C$."

Section 12.4

- When given Marginal Cost, Marginal Revenue, and Marginal Profit functions, be able to find the Cost, Revenue, and Profit functions, respectively. Also be able to calculate the constant of integration C using the given information.

Section 13.1

- Be able to interpret and calculate a sum using Sigma notation.
- Know how to approximate the area under a curve using a given number of subintervals with a right-hand approximation.
- Be able to calculate the exact area under a curve by first approximating it with n rectangles, then taking $n \to \infty$ using a right-hand approximation.

Section 13.2

- Be able to calculate definite integrals using the "shortcuts" from this section.
- Know how to use the definite integral to interpret word problems when given the instantaneous rate of change of a function, and asked for the exact change of the function on a given interval.

Section 13.3

- Be able to find the exact area between two curves on a given interval.
- Also be able to find the area bounded between two curves when no interval is given. Recall that this involves finding the x-values where the two curves intersect.
- Know how to calculate the average value of a function on a given interval.

Section 13.4

- When given a rate of continuous income flow, be able to calculate the Total Income for a given period of time.
- When given a rate of continuous income flow and an interest rate with interest that is compounded continuously, be able to calculate the Present Value and Future Value for a given period of time.

Section 13.7

- Be able to calculate definite integrals with at least one infinite bound, and determine whether they converge or diverge.

Section 14.1

- Be able to evaluate functions of 2 or more variables.
- Know how to find the domain of functions of 2 or more variables.

Section 14.2

- Be able to find partial derivatives of functions of 2 or more variables.
- Know how to calculate second partial derivatives of functions of multiple variables.

Sample Problems

1. Calculate each of the following integrals:

   (a) $\displaystyle\int \left(x^2 + \frac{3}{x^4} - \sqrt[7]{x}\right) dx$

   (b) $\displaystyle\int t\left(4t^2 - 5\right)^7 dt$

   (c) $\displaystyle\int \frac{x^3 + 1}{(x^4 + 4x)^8}\, dx$

   (d) $\displaystyle\int \frac{x^3 + 1}{x^4 + 4x}\, dx$

   (e) $\displaystyle\int y^3\, e^{3y^4 - 2}\, dy$

   (f) $\displaystyle\int \frac{dx}{x(\ln x)^3}$

   (g) $\displaystyle\int e^{\ln z^5}\, dz$

(h) $\int \ln e^{-5x^3/2+6}\,dx$

2. The marginal revenue function for a product is $\overline{MR} = \frac{x}{\sqrt{x^2+25}}$. Find the total revenue function.
3. #39–51 odds from Section 12.1.
4. #43–51 odds from Section 12.2.
5. #43–51 odds from Section 12.3.
6. #1–11 odds from Section 12.4.
7. Calculate each of the following sums:

   (a) $\sum_{k=1}^{5} 2k$

   (b) $\sum_{i=1}^{4} i^3$

   (c) $\sum_{j=2}^{3} (j^2 - 2)$

8. Approximate the area under $y = x^2 + 3$ from $x = 1$ to $x = 2$ using 4 subintervals and a right-hand approximation.
9. Calculate the exact area under the curve $y = x^2 + 3$ from $x = 1$ to $x = 2$ using the right-hand approximation.
10. Calculate each of the following definite integrals:

   (a) $\int_2^5 \left(x^2 - 3x + 5\right) dx$

   (b) $\int_{-2}^{1} \frac{dx}{(x-3)^2}$

   (c) $\int_0^{\sqrt{\ln 5}} xe^{x^2}\,dx$

   (d) $\int_3^4 \frac{dx}{x}$

   (e) $\int_0^{\infty} xe^{-x^2}\,dx$

   (f) $\int_{-\infty}^{-2} \frac{dx}{x^3}$

   (g) $\int_2^{\infty} \frac{dx}{x \ln x}$

   (h) $\int_1^{\infty} \frac{dx}{\sqrt{x}}$

11. Calculate the area bounded by the following sets of curves:
    (a) $f(x) = 2x^2$, $g(x) = -x - 1$, $x = 1$, and $x = 4$.
    (b) $f(x) = x^2 - 8$ and $g(x) = -x^2$.
    (c) $f(x) = 16x$ and $g(x) = x^3$.
12. Calculate the average value of $f(x) = 6x^2 - 4x + 5$ on the interval $[1, 4]$.
13. #1–15 odds from Section 13.4.
14. Given the function $f(x, y) = \ln(xy) + x^2y - 3xy$, find each of the following:
    (a) $f(\frac{1}{2}, 2)$
    (b) $f(-e, -e^2)$
    (c) $f(0, 3)$
    (d) $\frac{\partial}{\partial x} f(x, y)$
    (e) $\frac{\partial}{\partial y} f(x, y)$
    (f) $\frac{\partial^2}{\partial x^2} f(x, y)$
    (g) $\frac{\partial^2}{\partial y^2} f(x, y)$
    (h) $\frac{\partial^2}{\partial x \partial y} f(x, y)$
    (i) $\frac{\partial^2}{\partial y \partial x} f(x, y)$
    (j) What is the domain of f?