Study Guide for Exam 2
Math 1100-4

Formulas to Know:

- Average Cost: $\bar{C} = \dfrac{C(x)}{x}$

- $P = R - C$

- Logarithmic formulas and properties

- Exponential formulas and properties

- Area of a circle: $A = \pi r^2$

- Volume of sphere: $V = \frac{4}{3}\pi r^3$

- Pythagorean theorem: $a^2 + b^2 = c^2$

- Elasticity of demand: $\eta = -\dfrac{p}{q} \cdot \dfrac{dq}{dp}\bigg|_{(q_A, p_A)}$

Derivative Formulas:

- $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$

- $\dfrac{d}{dx}\ln g(x) = \dfrac{g'(x)}{g(x)}$

- $\dfrac{d}{dx}e^x = e^x$

- $\dfrac{d}{dx}e^{g(x)} = g'(x) \cdot e^{g(x)}$

- $\dfrac{d}{dx}a^{g(x)} = g'(x)\ln a \cdot a^{g(x)}$

Section 10.3

- Be able to find the absolute extrema for a continuous function on a closed interval.

- Be able to find extrema of Profit, Revenue, and Cost functions.

- When given a cost function, be able to find the minimum of the average cost function.

Section 10.4

- Be able to find absolute extrema for a function described in a word problem. Note that for some word problems, the function is given, and for others you will need to write the function.

- Be able to carefully read word problems to determine if you are looking for where the minimum/maximum occurs, what the minimum/maximum is, or both.

Section 10.5

- Be able to calculate both vertical and horizontal asymptotes of a rational function, and use the information when sketching a graph.

- Be able to find the x-intercept(s) and y-intercept of a rational function, and use this information when sketching a graph.

- Know how to find the relative extrema of a rational function, where it is increasing an decreasing, and information on concavity, and use this information when sketching a graph.

Section 11.1

- Know the properties of logarithms.

- Know how to take derivatives of logarithmic functions.

- Be able to combine other derivative formulas with the formula for the derivative of logarithmic functions.

Section 11.2

- Know how to take derivatives of exponential functions.

- Be able to combine derivative formulas with the formula for the derivative of exponential functions.

Section 11.3

- Be able to recognize when techniques from implicit differentiation are needed.

- Know how to find $\frac{dy}{dx}$ when an equation relating y to x is given, and be able to use this to calculate the instantaneous rate of change at a given point.

Section 11.4

- Be able to differentiate both sides of a given equation with respect to time t, even when t is not a given variable in the original equation.

- Be able to interpret and solve word problems involving related rate questions.

Section 11.5

- Be able to calculate the elasticity of demand.

- Be able to interpret the elasticity of demand to determine how a price increase will impact the demand of a product, and how a price change will impact the revenue.

Sample Problems

1. Find the absolute maxima and minima for $f(x) = x^3 - 2x^2 + x - 3$ on the interval $[-1, 2]$.

2. The total revenue function for a product is $R(x) = 5600x - 16x^2 - 2x^3$. Find the maximum revenue.

3. The total cost function for a product is $C(x) = 2500 + x^2$. Producing how many units, x, will result in a minimum average cost? Find the minimum average cost.

4. Three equal rectangular lots are enclosed by fencing the perimeter of a rectangular lot and then putting two fences across the middle. If each lot is to contain

600 square feet, what is the minimum amount of fencing needed to enclose the lots, including the fences across the middle?

5. #32 from Section 10.4.

6. For each of the following functions, find any horizontal and vertical asymptotes, intercept(s), and use information from the first derivative to sketch the graph.

(a) $f(x) = \dfrac{2x - 5}{x + 3}$

(b) $y = \dfrac{x^2 + 3}{x^2}$

7. #39 from Section 10.5.

8. Find the derivative of each of the following functions:

(a) $f(x) = 3 \ln x^2$

(b) $y = 3(\ln x)^2$

(c) $y = \ln(x^3 - 4)$

(d) $f(z) = \dfrac{\ln z^2 + 2z}{4}$

(e) $g(x) = \dfrac{x}{\ln x}$

(f) $y = \log_6 x^3$

(g) $y = e^{2x}$

(h) $f(x) = 4e^{x^2+1} + x$

(i) $g(x) = xe^x$

(j) $h(z) = e^{z^3} + (e^z)^3$

(k) $y = 7^{x^2}$

(l) $s(t) = \ln(e^t + 1)$

9. #51 from Section 11.2.

10. Calculate $\frac{dy}{dx}$ for each of the following equations:

(a) $x^2 - y^2 + 3 = 0$

(b) $e^y = x + 2x^2$

(c) $x^3 - y^2 + y^3 - x^2 + y = 7$

(d) $(x + y)^2 = 3x^2y$

11. Find the slope of the line tangent to the graph of $x^2 + y^3 + x = 1$ at the point $(0, 1)$.

12. #19 from Section 11.4.

13. #29-35 odds from Section 11.4.

14. #1-9 odds from Section 11.5.