Review Problems

1. a. $\lim_{x\to 4} x^3-2x+2 = 4^3-2\cdot 4+2 = 64-6 = \boxed{58}$

b. $\lim_{x\to 5} \dfrac{x^2-7x+10}{x^2-25} = \dfrac{(x-5)(x-2)}{(x-5)(x+5)} = \dfrac{5-2}{5+5} = \boxed{\dfrac{3}{10}}$

c. $\lim_{x\to -2} \dfrac{x^2+x+2}{x^2-4x-12} = \boxed{\text{DNE}}$ $\left(\dfrac{4-2+2}{4+8-12} = \dfrac{4}{0}\right)$

d. $\lim_{x\to 6} \begin{cases} x-6 & x\le 6 \\ x^2+36 & x>6 \end{cases}$ $\boxed{\text{DNE}}$ $\lim_{x\to 6^-} = 0$, $\lim_{x\to 6^+} = 72$

e. $\lim_{x\to 6^+} \begin{cases} x-6 & x\le 6 \\ x^2+36 & x>6 \end{cases} = 6^2+36 = \boxed{72}$

f. $\lim_{x\to\infty} \dfrac{2x^3-5x^2+x-6}{3x^3+8x+4} = \boxed{\dfrac{2}{3}}$

2. $\lim_{x\to\infty} f(x) = \dfrac{2}{3} \to \boxed{y=\dfrac{2}{3}}$ is the horiz. asymp.

3. Continuity: $x^2-16=0$ → $f(x)$ discontinuous at $x=\pm 4$

4. $y = \dfrac{x(x^2+3x-4)}{(x-4)(x+4)} = \dfrac{x(x+4)(x-1)}{(x-4)(x+4)}$ vert asymptote is

~~x~~ $=4$ (not ~~x~~ $=-4$)

5. $\dfrac{f(b)-f(a)}{b-a} = \dfrac{2\cdot 2^3-4\cdot 2^2+1-(2(-1)^3-4(-1)^2+1)}{2-(-1)} = \dfrac{1-(-5)}{3}$

$= \dfrac{6}{3} = \boxed{2}$

6. a. $f'(x) \lim_{h\to 0} \dfrac{2(x+h)^2-3(x+h)+4-(2x^2-3x+4)}{h}$

$= \lim_{h\to 0} \dfrac{2x^2+4xh+2h^2-3x-3h+4-2x^2+3x-4}{h}$

$= \lim_{h\to 0} (4x+2h-3) = \boxed{4x-3}$

b. $f'(2) = 4\cdot 2-3 = \boxed{5}$

7. a. $f(x) = \pi^4 \rightarrow \boxed{f'(x) = 0}$ constant

b. $y = 2x^3 - 3x^{-5} + 3$, $y' = \boxed{6x^2 + 15x^{-6}}$

c. $y = \frac{5}{6}(1-x^3)^{1/2}$, $y' = \frac{5}{6} \cdot \frac{1}{2}(1-x^3)^{-1/2} \cdot (-3x^2) = \boxed{-\frac{5}{4}x^2(1-x^3)^{-1/2}}$

d. $g(t) = \frac{1-t}{(1+t)^2}$, $g'(t) = \frac{(1+t)^2(-1) - (1-t)\cdot 2(1+t)}{(1+t)^4}$

$= \frac{(1+t)(-1) - 2(1-t)}{(1+t)^3} = \frac{-1-t-2+2t}{(1+t)^3} = \boxed{\frac{t-3}{(1+t)^3}}$

e. $h(z) = (z+1)^2(z^2-z+2)^3$

$h'(z) = \boxed{(z+1)^2 \cdot 3(z^2-z+2)^2(2z-1) + (z^2-z+2)^3 \cdot 2(z+1)}$

8. $y = x^2 + 2x^{-1}$

$y' = 2x - 2x^{-2}$ @ $x = 3 \rightarrow 6 - \frac{2}{9} = \frac{54-2}{9} = \boxed{\frac{52}{9}}$

9. a. $f(x) = 2x^7 - 4x^{-5}$

$f'(x) = 14x^6 + 20x^{-6}$

$f''(x) = \boxed{84x^5 - 120x^{-7}}$

b. $y = (2x-6)^5$

$y' = 10(2x-6)^4$

$y'' = 80(2x-6)^3$

$\boxed{y''' = 480(2x-6)^2}$

10. a. $MP = 22 - 0.002x$

b. $P'(10) = 22 - 0.02 = 21.98 \rightarrow$ profit from sale of 11th unit is $\approx \$21.98$

c. $P(11) - P(10) = (22 \cdot 11 - 0.001 \cdot 11^2 - 7) - (22 \cdot 10 - 0.001 \cdot 10^2 - 7)$

$= 234.879 - 212.9$

$= \$21.979$ exact change in profit

d. $P'' = -0.002 \rightarrow$

11. $y = 2x^3 - 3x^2 - 1$

a. y-int $= -1$ (when $x = 0$)

b. $y' = 6x^2 - 6x = 0$

$6x(x-1) = 0$

$x = 0, 1$

c. Increasing: $(-\infty, 0) \cup (1, \infty)$

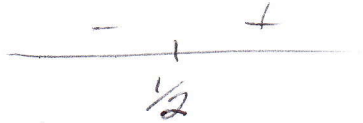
Decreasing: $(0, 1)$

d. Rel max of -1 @ $x = 0$

Rel. min of $2 - 3 - 1 = -2$ @ $x = 1$

e. $y'' = 12x - 6$

$x = \frac{1}{2}$



Concave down $(-\infty, \frac{1}{2})$

Concave up $(\frac{1}{2}, \infty)$

Pt of inflection $(\frac{1}{2}, -\frac{3}{2})$

f.