Section 9.9, Applications of Derivatives in Business and Economics

If $R = R(x)$ is the revenue function for a product, then the **marginal revenue function** is $\overline{MR} = R'(x)$.

Example

The total revenue function for a kind of t-shirt is $R(x) = 16x - 0.01x^2$, where R is in dollars and x is the number of t-shirts sold. Find the following:

1. Find the revenue when 40 units are sold.

   $$R(40) = 16 \cdot 40 - 0.01 \cdot 40^2 = 624$$

2. Find the marginal revenue function.

   $$\overline{MR} = R'(x) = 16 - 0.02x$$

3. Find the marginal revenue at $x = 40$. What does the predict about the sale of the next unit?

   $$R'(40) = 16 - 0.02 \cdot 40 = 15.20$$

   Since the derivative represents the rate of change, the sale of the next t-shirt will increase revenue by approximately \$15.20.

4. Find $R(41) - R(40)$. What does this quantity represent?

   $$R(41) - R(40) = 16 \cdot 41 - 0.01 \cdot 41^2 - 624 = 639.19 - 624 = 15.19$$

   This is the exact change in revenue between the 40th and 41st t-shirts.

If $C = C(x)$ is the cost function for a product, then its derivative, $\overline{MC} = C'(x)$, is the **marginal cost function**.

Example

Let $C(x) = 4x^2 - 16x + 40$. Find the marginal cost.

$$\overline{MC} = 8x - 16$$

If $P = P(x)$ is the profit function for a product, then its derivative, $\overline{MP} = P'(x)$, is called the **marginal profit function**.

Example

Let $P(x) = 7x - 45$. Find the marginal profit.

$$\overline{MP} = 7$$

Note that this means that the profit increases by \$7 for every unit sold.

Since $P = R - C$, $\overline{MP} = \overline{MR} - \overline{MC}$ when the functions represent the profit, revenue, and cost of the same product.