

Section 9.7, Using Derivative Formulas

In this section, no new formulas will be introduced. We will be practicing using multiple rules in one problem.

To decide which rules to use, look at the general structure of the function. For example, if there is multiplication, the Product Rule will be needed. With fractions, the Quotient Rule will be needed. For an involved function raised to a power, the General Power Rule is necessary. To decide the order in which to use the rules, order of operations (PEMDAS) will help. The last step you would use in evaluating the function tells which rule to use first.

Examples Find the derivative of each of the following functions:

1. $y = x^2\sqrt{4-x} = x^2 \cdot (4-x)^{1/2}$

Since there is a product, as well as a quantity raised to a power, we will need both the Product and General Power Rules. Since multiplication would be used last, we will start with the Product Rule, and use the Power Rule when we need the derivative of the factor with the power.

$$\begin{aligned}\frac{dy}{dx} &= x^2 \cdot \frac{1}{2}(4-x)^{-1/2}(-1) + (4-x)^{1/2} \cdot 2x \\ &= -\frac{1}{2}x^2(4-x)^{-1/2} + 2x(4-x)^{1/2}\end{aligned}$$

2. $f(t) = \frac{10t-1}{(t^2+2)^3}$

There is a fraction, and a quantity raised to a power within that fraction, so we will start by using the Quotient Rule, and use the General Power Rule when the derivative of the denominator is needed.

$$\begin{aligned}f'(t) &= \frac{(t^2+2)^3 \cdot 10 - (10t-1) \cdot 3(t^2+2)^2 \cdot 2t}{((t^2+2)^3)^2} \\ &= \frac{(t^2+2)^3 \cdot 10 - 6t(10t-1)(t^2+2)^2}{(t^2+2)^6} \\ &= \frac{(t^2+2) \cdot 10 - 6t(10t-1)}{(t^2+2)^4} \\ &\quad \text{since the } (t^2+2)^2 \text{ can be factored out of the numerator and denominator.} \\ &= \frac{10t^2+20-60t^2+6t}{(t^2+2)^4} = \frac{-50t^2+6t+20}{(t^2+2)^4}\end{aligned}$$

Note: When the denominator is a quantity raised to an exponent, some powers of the denominator will cancel out.

3. $g(t) = [(t+9)(t^2-4)]^5$

Since there is a product within a complicated exponent, we will need to use the Product Rule when taking the derivative of the “inside of the parentheses” with the General Power Rule.

$$\begin{aligned}g'(t) &= 5[(t+9)(t^2-4)]^4 \cdot [(t+9) \cdot 2t + (t^2-4) \cdot 1] \\ &= 5[(t+9)(t^2-4)]^4 \cdot [3t^2+18t-4]\end{aligned}$$