Section 9.4, Derivative Formulas

In this section and the few that follow, we will learn several "shortcut" rules to calculate derivatives. (Note: The limit definition of the derivative may still show up now and again throughout the semester, such as on exams and when we consider more complicated functions.)

Power Rule: If $f(x) = x^n$, for any real number n, then $f'(x) = nx^{n-1}$.

Examples

Find derivatives for each of the following functions:

1. $f(x) = x^2$

   Here $n = 2$, so $f'(x) = 2x^{2-1} = 2x$.

2. $f(x) = x$

   Rewrite the function as $f(x) = x^1$, so $f'(x) = 1x^0 = 1$ (since $x^0 = 1$).

3. $f(x) = \frac{1}{x}$

   We can rewrite this as $f(x) = x^{-1}$, so $f'(x) = -1 \cdot x^{-1-1} = -x^{-2}$. (We will learn how to find derivatives of more complicated fractions in the next section.)

4. $g(x) = \sqrt{x}$

   We can rewrite this as $g(x) = x^{1/2}$, so $g'(x) = \frac{1}{2}x^{-1/2}$.

Constant Function Rule: If $f(x) = c$, where c is a constant, then $f'(x) = 0$.

Example

Let $y = 9$. Then, $y' = 0$.

Constant Coefficient Rule: If $f(x) = c \cdot g(x)$, where c is a constant, then $f'(x) = c \cdot g'(x)$.

Example

Assume that $h(z) = 2z^4$. Then, $h'(z) = 2 \cdot 4z^3 = 8z^3$, by using the power rule for the z^4 part.

Sum and Difference Rules: If $f(x) = u(x) \pm v(x)$, then $f'(x) = u'(x) \pm v'(x)$

Examples

1. Find the derivatives of each of the following functions:

   (a) $f(x) = 2x^3 + 2x - 4$

   Combining all of the above rules, $f'(x) = 2 \cdot 3x^2 + 2 \cdot 1 + 0 = 6x^2 + 2$

   (b) $g(x) = 7x^8 - \frac{8}{x^3} + 2 = 7x^8 - 8x^{-3} + 2$

   $g'(x) = 7 \cdot 8x^7 - 8(-3)x^{-4} + 0 = 56x^7 + 24x^{-4}$

2. Find the point(s) at which the graph of $f(x) = 4x^3 - 3x + 4$ has a horizontal tangent line.

   Recall from Algebra that horizontal lines have a slope of 0. From the last section, we know that the slope of the tangent line at a given point equals the derivative at that point. So, this question is asking us to find the points where the x-values that give $f'(x) = 0$.

   First, we must find the derivative of the function:

   $$f'(x) = 12x^2 - 3$$

Now, we set $f'(x) = 0$ and solve for x:

$$0 = 12x^2 - 3 = 3(4x^2 - 1) = 3(2x - 1)(2x + 1)$$
$$x = \frac{1}{2}, -\frac{1}{2}$$

Since we were asked for points (as opposed to just the x-values), we still need to find the value of the function (not the derivative) at the x-values we just found:

$$f\left(\frac{1}{2}\right) = 3$$
$$f\left(-\frac{1}{2}\right) = 5,$$

so the points are $(\frac{1}{2}, 3)$ and $(-\frac{1}{2}, 5)$.