Section 14.1, Functions of Two or More Variables

So far, we have looked at functions of only one variable, typically we have used x. For this chapter, we will be looking at functions of two or more variables. For example, $f(x, y) = 3x^2y + 4x + y + 1$ or $z = 2x^2e^y + \ln xy$. In this section, we will be evaluating these functions and finding their domains.

Examples

1. Let $f(x, y) = 4x^2y^2 - xy + e^y$. Find $f(2, 1)$.

$$f(2, 1) = 4 \cdot 2^2 \cdot 1^2 - 2 \cdot 1 + e^1 = 14 + e$$

2. Evaluate $z = \ln xy - x^2y + y$ at the point $(4, \frac{1}{4})$.

$$z = \ln\left(4 \cdot \frac{1}{4}\right) - 4^2 \cdot \frac{1}{4} + \frac{1}{4} = 0 - 4 + \frac{1}{4} = -\frac{15}{4}$$

3. We also did #26 from the book.

The **domain** of a function is the set of all possible values of the independent variables.

Examples

Find the domain of each of the following functions

1. $z = \sqrt{x - y}$

   Since square roots are not defined for negative numbers, we need $x - y \geq 0$, or $x \geq y$.

2. $z = \frac{x^2+y^2}{x}$

   We cannot divide by 0, so $x \neq 0$, but y can be any real number.

3. $z = 2x - 3y$

   This function has a linear form, so x and y can be any real number.

4. $f(x, y) = \ln(xy)$

   We cannot take the logarithm of a negative number or zero, so the domain is all values of x and y with $xy > 0$.