Section 13.4, Applications of Definite Integrals in Business and Economics

In Math 1090, you learned about the "present value" and "future value" of investments (the term "annuity" was used). Typically, money was being deposited (or withdrawn, for present value problems) on a regular schedule, and the interest involved was compounded on that same regular schedule. For example, when setting aside \$500 per month for a retirement plan with interest compounded monthly, you would be interested in the future value (i.e. the value later, such as when you plan to retire). An example involving present value would be to determine how much money you will need to have when you retire, if you want to ensure that you can withdraw \$4000 per month from an account with fixed interest compounded monthly for 40 years after you retire.

Now, instead of looking at what happens when money is deposited in set intervals, we will look at what happens when income is continuously coming in with a "continuous income flow." Continuous income flows are helpful for determining income in situations where the income varies over time. The book uses the example of income from pumping oil from an oil field. Since the pump and oil field both wear out over time, the function $f(t)$ would be decreasing. For present and future value problems, interest will be compounded continuously. For most of these examples, a calculator will be necessary to get an answer that makes sense in the context of the question (i.e. an actual dollar amount)

1 Total Income

If $f(t)$ is the rate of continuous income flow, the **total income** for the first k years is

$$TI = \int_0^k f(t)\,dt$$

2 Present Value

If $f(t)$ is the rate of continuous income flow earning interest at a rate r, compounded continuously, then the **present value** of the continuous income stream is:

$$PV = \int_0^k f(t)e^{-rt}\,dt$$

where $t = 0$ to $t = k$ is the time interval.

3 Future Value

If $f(t)$ is the rate of continuous income flow for k years earning interest at a rate of r, compounded continuously, then the **future value** of the continuous income stream is

$$FV = e^{rk}\int_0^k f(t)e^{-rt}\,dt = e^{rk}\cdot PV$$

Examples

We did #3, 11, and 13 from the book.