

Section 12.4, Applications of the Indefinite Integral in Business and Economics

Examples

1. The monthly marginal cost for a product is $\overline{MC} = x + 30$. If fixed costs are \$50, find the total cost function for the month.

We will use that the cost is the antiderivative of the marginal cost, and then used the fixed cost to find the value of the constant C .

$$C(x) = \int \overline{MC} \, dx = \int (x + 30) \, dx = \frac{x^2}{2} + 30x + C$$

$$C(0) = 50 = C$$

$$C(x) = \frac{x^2}{2} + 30x + 50$$

2. The marginal revenue function for a product is $\overline{MR} = 44 - 5x$. The marginal cost is $\overline{MC} = 3x + 20$, and the cost of producing 80 units is \$11400.

- (a) Find the profit function.

Here, we are given information about revenue and cost, but are asked about profit, so we will need to find the revenue and cost functions, and then use $P = R - C$ to find the profit function

$$R(x) = \int (44 - 5x) \, dx = 44x - \frac{5}{2}x^2 + C$$

$$R(0) = 0 \text{ gives us that } C = 0, \text{ so } R(x) = 44x - \frac{5}{2}x^2$$

$$C(x) = \int (3x + 20) \, dx = \frac{3}{2}x^2 + 20x + C$$

$$C(80) = 11400 = \frac{3}{2} \cdot 80^2 + 20 \cdot 80 + C = 9600 + 1600 + C = 11200 + C$$

$$C = 200$$

$$C(x) = \frac{3}{2}x^2 + 20x + 200$$

$$P(x) = 44x - \frac{5}{2}x^2 - \left(\frac{3}{2}x^2 + 20x + 200 \right) = 24x - 4x^2 - 200$$

- (b) Find the profit or loss from selling 100 units.

$$P(100) = 24 \cdot 100 - 4 \cdot 100^2 - 200 = -37800, \text{ so there is a loss of } \$37800.$$

3. The average cost of a product changes at the rate $\overline{C}'(x) = -6x^{-2} + \frac{1}{6}$. The average cost of 6 units is 10. Find the average cost of producing 12 units.

First, we need the average cost function, then we can evaluate it at $x = 12$.

$$\bar{C}(x) = \int \left(-6x^{-2} + \frac{1}{6} \right) dx = 6x^{-1} + \frac{x}{6} + C$$

$$\bar{C}(6) = 10 = 6 \cdot 6^{-1} + \frac{6}{6} + C = 2 + C$$

$$C = 8$$

$$\bar{C}(x) = 6x^{-1} + \frac{x}{6} + 8$$

$$\bar{C}(12) = 6 \cdot 12^{-1} + \frac{12}{6} + 8 = \frac{1}{2} + 2 + 8 = 10.5$$