Section 12.2, The Power Rule

(General) Power Rule for Integration If $n \neq -1$,

$$\int [u(x)]^n u'(x)\,dx = \frac{[u(x)]^{n+1}}{n+1} + C$$
$$\text{or } \int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

Examples

1. $\int (x^2+1)^5 2x\,dx = \frac{(x^2+1)^6}{6} + C$
2. $\int (3x^3 - 9x^2 + 14)^{10}(9x^2 - 18x)\,dx = \frac{(3x^3-9x^2+14)^{11}}{11} + C$
3. $\int (2x^2+4)^3 x\,dx = \frac{1}{4}\int (2x^2+4)^3 4x\,dx = \frac{(2x^2+4)^4}{16} + C$
4. $\int \sqrt[3]{2x+1}\,dx = \frac{1}{2}\int 2(2x+1)^{1/3}\,dx = \frac{1}{2}\frac{3}{4}(2x+1)^{4/3} + C = \frac{3}{8}(2x+1)^{4/3} + C$
5. $\int \frac{2x^3-1}{(x^4-2x)^5}\,dx = \frac{1}{2}\int u^{-5}\,du = -\frac{1}{8}u^{-4} + C = -\frac{1}{8}(x^4-2x)^{-4} + C$, where we used $u(x) = x^4 - 2x$.
6. A product's marginal revenue function is $\overline{MR} = 60,000 - \frac{40,000}{(10+x)^2}$. Find the total revenue function.

$$\begin{aligned} R(x) &= \int \overline{MR}\,dx = \int (60,000 - 40,000(10+x)^{-2}\,dx \\ &= 60,000x + 40,000(10+x)^{-1} + C \\ 0 = R(0) &= 0 + 40,000 \cdot 10^{-1} + C \\ C &= -4000 \\ R(x) &= 60,000x + 40,000(10+x)^{-1} - 4000 \end{aligned}$$