Section 12.1, The Indefinite Integral

1 Integrals

Given a derivative, $f'(x)$, the process of finding the function $f(x)$ is called **antidifferentiation** or **integration**. The most general function $f(x)$ is called the **integral** or **indefinite integral** of $f'(x)$, and we write $\int f'(x)\,dx = f(x)$.

Examples

Find each of the following:

1. $\int 4x^3\,dx = x^4 + C$, where C represents a general constant.
2. $\int x^6\,dx = \frac{x^7}{7} + C$

Powers of x Formula: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$.

Examples

Calculate and simplify each of the following:

1. $\int x^2\,dx = \frac{x^3}{3} + C$
2. $\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{2x^{3/2}}{3} + C$
3. $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$

2 Properties

There are many general properties of integrals. Let c and C represent constants:

1. $\int cu(x)\,dx = c\int u(x)\,dx$
2. $\int (u(x) \pm \nu(x))\,dx = \int u(x)\,dx \pm \int \nu(x)\,dx$
3. $\int 1\,dx = x + C$
4. $\int 0\,dx = C$

Examples

1. $\int (3 + 2x^2)\,dx = 3x + \frac{2}{3}x^3 + C$
2. $\int (4 - \frac{1}{\sqrt[3]{x^2}})\,dx = \int (4 - x^{-2/3})\,dx = 4x - 3x^{1/3} + C$
3. $\int (\frac{3}{x^9} - \frac{16}{\sqrt[5]{x}})\,dx = \int (3x^{-9} - 16x^{-1/5})\,dx = -\frac{3}{8}x^{-8} - 20x^{4/5} + C$
4. The marginal revenue for a product is $\overline{MR} = -0.5x + 60$. Find $R(x)$.

   $R(x) = \int (-0.5x + 60)\,dx = -0.25x^2 + 60x + C$. Using that $R(0) = 0$, we see that $C = 0$, so $R(x) = -0.25x^2 + 60x$.