Section 11.2, Derivatives of Exponential Functions

If $y = e^x$, then $y' = e^x$. By the chain rule, if $y = e^{g(x)}$, then $y' = e^{g(x)} \cdot g'(x)$.

Examples

1. $f(x) = 6e^x$

   $$f'(x) = 6e^x$$

2. $g(x) = e^{x^2}$

   $$g'(x) = 2xe^{x^2}$$

3. $y = 3e^{x^4-1}$

   $$y' = 12x^3e^{x^4-1}$$

4. $y = x^2e^{x^2}$

   Here, we have a product, so the product rule is necessary:

   $$y' = 2xe^{x^2} + 2x^3e^{x^2}$$

5. $h(z) = \ln e^{z^4}$

   By properties of logarithms, we can simply this function to $h(z) = z^4$, so

   $$h'(z) = 4z^3$$

6. $f(x) = \frac{2+e^x}{e^{2x}} = 2e^{-2x} + e^{-x}$

   $$f'(x) = -4e^{-2x} - e^{-x}$$

7. $y = \ln(e^{z^4} + 1)$

   $$y' = \frac{4z^3e^{z^4}}{e^{z^4} + 1}$$

We can generalize the derivative of $y = e^x$ to $y = a^x$ if $a > 0$ and $a \neq 1$. If $y = a^x$, then $y' = a^x \ln a$. If $y = a^{g(x)}$, then $y' = a^{g(x)}g'(x) \ln a$.

This holds because we can rewrite y as $y = a^x = e^{\ln a^x} = e^{x \ln a}$, then use the formula for the derivative of the exponential function of base e.

Examples

1. $y = 5^x$

   $$y' = 5^x \ln 5$$

2. $y = 10^{x^3}$

   $$y' = 10^{x^3} \cdot 3x^2 \ln 10$$