

Section 10.5, Rational Functions: More Curve Sketching

Here is a list of things that may help when graphing functions:

1. Horizontal and/or vertical asymptotes (sketch these using dashed lines)
2. The y -intercept and x -intercept(s)
3. Relative extrema
4. Concavity

Examples Find any horizontal and vertical asymptotes, intercepts, and use information from the first derivative to sketch the graph of the following:

1. $y = \frac{2x+4}{x-2}$

Since the denominator is zero and the numerator is nonzero at $x = 2$, the line $x = 2$ is a vertical asymptote. To find the horizontal asymptote, we need to find the limit as $x \rightarrow \infty$:

$$\lim_{x \rightarrow \infty} \frac{2x+4}{x-2} = 2$$

Therefore, the horizontal asymptote is the line $y = 2$.

Now, we must find the intercepts:

To find the y -intercept, we need the value of y when $x = 0$:

$$\frac{2 \cdot 0 + 4}{0 - 2} = -2$$

To find the x -intercept(s), we need the value(s) of x that give $y = 0$:

$$\begin{aligned} 0 &= \frac{2x+4}{x-2} \\ 0 &= 2x+4 \\ x &= -2 \end{aligned}$$

Now, we need information from the first derivative. To find the derivative, we will use the quotient rule:

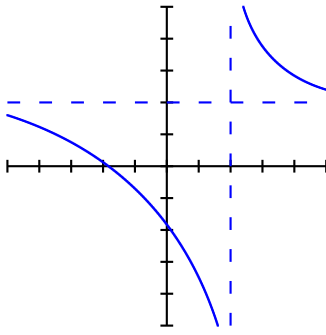
$$y' = \frac{(x-2) \cdot 2 - (2x+4) \cdot 1}{(x-2)^2} = -\frac{8}{(x-2)^2}$$

This is never zero, and is only undefined at our vertical asymptote, $x = 2$. Note that the derivative is negative whenever it is defined, so our function will always be decreasing when it is defined.

We will now find information on concavity from the second derivative:

$$y'' = \frac{16}{(x-2)^3}$$

This is undefined at $x = 2$, negative for $x < 2$, and positive for $x > 2$, so the graph of the function is concave down for $x < 2$ and concave up for $x > 2$.



2. $y = \frac{x^2 - 8}{x^2}$

The denominator is zero at $x = 0$, so there is a vertical asymptote at $x = 0$.

Taking the limit as $x \rightarrow \infty$, we get a horizontal asymptote of $y = 1$.

There is no y -intercept, since the function is undefined at $x = 0$.

To find the x -intercept(s),

$$0 = \frac{x^2 - 8}{x^2}$$

$$0 = x^2 - 8$$

$$x = \pm\sqrt{8} = \pm 2\sqrt{2} \approx \pm 2.828$$

Now, we need the derivative:

$$y' = \frac{x^2 \cdot 2x - (x^2 - 8) \cdot 2x}{x^4} = \frac{16}{x^3}$$

This is negative when $x < 0$ and positive for $x > 0$, so the function is decreasing for $x < 0$ and increasing for $x > 0$.

The second derivative is:

$$y'' = -48x^{-4}$$

which is undefined at $x = 0$, and is otherwise negative. So, for $x \neq 0$, the graph of the function is concave down.

