Solutions for Diagnostic Problems
Math 1100-4
Tuesday, January 10, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. This assignment is due by the end of class on Tuesday, January 10.

1. What was the last math class that you took? When did you take it?

2. What other math classes do you need to take?

3. What is your major?

4. Let $f(x) = \frac{x^2+x+3}{x+2}$. Find the following, if they exist. If the function is undefined at that point, state why:

   (a) $f(-1)$
   $$f(-1) = \frac{(-1)^2 + (-1) + 3}{(-1) + 2} = \frac{3}{1} = 3$$

   (b) $f(4)$
   $$f(4) = \frac{4^2 + 4 + 3}{4 + 2} = \frac{23}{6}$$

   (c) $f(-2)$
   Since the denominator is zero at $x = -2$, $f(-2)$ is undefined.

5. Factor the following expressions:

(a) $x^2 - 9$

$$x^2 - 9 = (x - 3)(x + 3)$$

(b) $x^2 + x - 6$

$$x^2 + x - 6 = (x + 3)(x - 2)$$

(c) $x^2 + 4x + 3$

$$x^2 + 4x + 3 = (x + 3)(x + 1)$$

6. Write the following expressions using exponents, leaving neither fractions nor radicals in your final answer:

(a) $\sqrt{x}$

$$\sqrt{x} = x^{1/2}$$

(b) $\frac{1}{y}$

$$\frac{1}{y} = y^{-1}$$

(c) $\frac{1}{\sqrt[5]{t^3 + 3}}$

$$\frac{1}{\sqrt[5]{t^3 + 3}} = (t^3 + 3)^{-1/5}$$

7. Simplify each of the following:

(a) $\frac{4(x+h)^2-4x^2}{h}$

$$\frac{4(x+h)^2-4x^2}{h} = \frac{4x^2+8xh+4h^2-4x^2}{h} = \frac{8xh+4h^2}{h}$$
$$= 8x + 4h \text{ for } h \neq 0.$$

(b) $(x^2+3x+1)(3x^2)+(2x+3)(x^3+1)$

$$(x^2+3x+1)(3x^2)+(2x+3)(x^3+1)$$
$$= 3x^4 + 9x^3 + 3x^2 + 2x^4 + 2x + 3x^3 + 3$$
$$= 5x^4 + 12x^3 + 3x^2 + 2x + 3$$

(c) $\frac{x^2-4}{x-2}$

$$\frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x+2$$

You should also have stated that this holds only when $x \neq 2$, but I didn't take off points for that technicality.

(d) $\frac{x(3x^2)-x^3(1)}{x^2}$

$$\frac{x(3x^2)-x^3(1)}{x^2} = \frac{3x^3-x^3}{x^2} = \frac{2x^3}{x^2} = 2x \text{ for } x \neq 0.$$

8. Evaluate each of these exponents and logarithms without the use of a calculator:

(a) 5^0

$5^0 = 1$ since any number (except 0) to the 0th power is 1.

(b) 6^1

$6^1 = 6$ since any number raised to the 1st power is itself.

(c) $\log_2 8$

$\log_2 8 = 3$ since $2^3 = 8$.

(d) $\log_3 \frac{1}{3}$

$\log_3 \frac{1}{3} = -1$ since $3^{-1} = \frac{1}{3}$.

9. Give the real solution(s) for x that satisfy the following equations (Hint: The Quadratic Formula might be helpful for one of the following.):

(a) $x^2 - x = 6$

$0 = x^2 - x - 6 = (x - 3)(x + 2)$
$x = 3, -2$

(b) $2x^2 - 4x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2}$$
$$= \frac{4 \pm \sqrt{40}}{4} = \frac{2 \pm \sqrt{10}}{2}$$