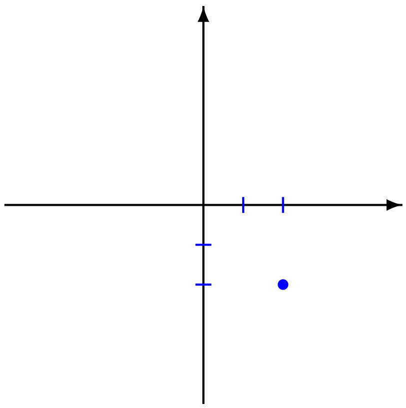
Quiz 8 Key

Math 1060–5

Friday, November 30, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. No calculators are allowed.

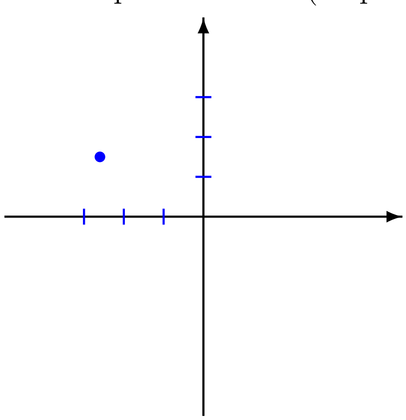
1. Find the trigonometric form of $z = 2 - 2i$. Give any angles in radians. Also graph the complex number. (13 points)



From formulas, we know that the modulus of the complex number is $r = \sqrt{a^2 + b^2} = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$. The angle θ satisfies the equation $\tan\theta = \frac{b}{a} = \frac{-2}{2} = -1$. Since the number is in Quadrant IV, we know that the angle must be $\theta = \frac{7\pi}{4}$. Therefore, the complex number in trigonometric form is

$$2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right).$$

2. Convert the complex number $z = 3\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ to standard form. Also graph the complex number. (13 points)



$$z = 3\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 3\left(\frac{-\sqrt{3}}{2} + i\frac{1}{2}\right) = -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$$

3. Perform each of the following operations and leave your results in trigonometric form with angles between 0 and 2π (or $0°$ and $360°$ when degrees are given.). (12 points each)

(a) $\left[\frac{3}{4}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]\left[4\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)\right]$

$$\begin{aligned}
&\left[\frac{3}{4}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]\left[4\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)\right]\\
&\quad=\frac{3}{4}\cdot 4\left[\cos\left(\frac{\pi}{3}+\frac{3\pi}{4}\right)+i\sin\left(\frac{\pi}{3}+\frac{3\pi}{4}\right)\right]\\
&\quad=3\left[\cos\frac{13\pi}{12}+i\sin\frac{13\pi}{12}\right]
\end{aligned}$$

(b) $\dfrac{2\left(\cos 120°+i\sin 120°\right)}{4\left(\cos 40°+i\sin 40°\right)}$

$$\begin{aligned}
\frac{2\left(\cos 120°+i\sin 120°\right)}{4\left(\cos 40°+i\sin 40°\right)} &=\frac{2}{4}\left[\cos(120°-40°)+i\sin(120°-40°)\right]\\
&=\frac{1}{2}\left[\cos 80°+i\sin 80°\right]
\end{aligned}$$