Quiz 6 Key
Math 1060–5
Friday, October 26, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. No calculators are allowed.

Formulas

$$\sin(u+\mathrm{v}) = \sin u\cos\mathrm{v} + \cos u\sin\mathrm{v} \qquad \sin 2u = 2\sin u\cos u$$
$$\sin(u-\mathrm{v}) = \sin u\cos\mathrm{v} - \cos u\sin\mathrm{v} \qquad \cos 2u = \cos^2 u - \sin^2 u$$
$$\cos(u+\mathrm{v}) = \cos u\cos\mathrm{v} - \sin u\sin\mathrm{v} \qquad = 2\cos^2 u - 1$$
$$\cos(u-\mathrm{v}) = \cos u\cos\mathrm{v} + \sin u\sin\mathrm{v} \qquad = 1 - 2\sin^2 u$$
$$\tan(u+\mathrm{v}) = \frac{\tan u + \tan\mathrm{v}}{1-\tan u\tan\mathrm{v}} \qquad \tan 2u = \frac{2\tan u}{1-\tan^2 u}$$
$$\tan(u-\mathrm{v}) = \frac{\tan u - \tan\mathrm{v}}{1+\tan u\tan\mathrm{v}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$
$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

1. Find the exact value of each of the following: (13 points each)

(a) $\sin\dfrac{11\pi}{12}$ (Hint: $\dfrac{11\pi}{12} = \dfrac{3\pi}{4} + \dfrac{\pi}{6}$.) (#11 from 5.4)

$$\begin{aligned}\sin\frac{11\pi}{12} &= \sin\left(\frac{3\pi}{4}+\frac{\pi}{6}\right)\\ &= \sin\frac{3\pi}{4}\cos\frac{\pi}{6} + \cos\frac{3\pi}{4}\sin\frac{\pi}{6}\\ &= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{-\sqrt{2}}{2}\cdot\frac{1}{2}\\ &= \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

(b) $\cos\dfrac{17\pi}{12}$ (Hint: $\dfrac{17\pi}{12} = \dfrac{9\pi}{4} - \dfrac{5\pi}{6}$.) (#13 from 5.4)

$$\begin{aligned}\cos\frac{17\pi}{12} &= \cos\left(\frac{9\pi}{4}-\frac{5\pi}{6}\right)\\ &= \cos\frac{9\pi}{4}\cos\frac{5\pi}{6} + \sin\frac{9\pi}{4}\sin\frac{5\pi}{6}\\ &= \frac{\sqrt{2}}{2}\cdot\frac{-\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\\ &= \frac{-\sqrt{6}+\sqrt{2}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}$$

(c) $\tan \dfrac{\pi}{12}$

$$\begin{aligned}\tan \frac{\pi}{12} &= \tan \frac{\pi/6}{2} \\ &= \frac{1-\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}} \\ &= \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} \\ &= 2-\sqrt{3}\end{aligned}$$

You could also reach the same answer by using that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$.

2. Let $\sin u = \dfrac{4}{5}$, where u is an angle in Quadrant II. Find $\sin 2u$. (11 points)

Note that $\cos u = -\frac{3}{5}$. You can get this by either using the Pythagorean Theorem or sketching a triangle. Then,

$$\begin{aligned}\sin 2u &= 2\sin u \cos u \\ &= 2 \cdot \frac{4}{5} \cdot \frac{-3}{5} = -\frac{24}{25}\end{aligned}$$