

Quiz 5

Key

Math 1060–5

Friday, October 19, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. No calculators are allowed.

1. Simplify the following expression: (15 points) (#35 from 5.1)

$$\sec \alpha \cdot \frac{\sin \alpha}{\tan \alpha}$$

$$\begin{aligned} \sec \alpha \cdot \frac{\sin \alpha}{\tan \alpha} &= \frac{1}{\cos \alpha} \cdot \frac{\sin \alpha}{\frac{\sin \alpha}{\cos \alpha}} \\ &= \frac{1}{\cos \alpha} \cdot \sin \alpha \cdot \frac{\cos \alpha}{\sin \alpha} = 1 \end{aligned}$$

2. Verify the following identity: (15 points) (#21 from 5.2)

$$\frac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$$

$$\begin{aligned} \frac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 &= \frac{\cos \theta \cdot \frac{\cos \theta}{\sin \theta}}{1 - \sin \theta} - 1 \\ &= \frac{\cos^2 \theta}{\sin \theta(1 - \sin \theta)} - 1 \\ &= \frac{\cos^2 \theta}{\sin \theta(1 - \sin \theta)} - \frac{\sin \theta(1 - \sin \theta)}{\sin \theta(1 - \sin \theta)} \\ &= \frac{\cos^2 \theta - \sin \theta + \sin^2 \theta}{\sin \theta(1 - \sin \theta)} \\ &= \frac{1 - \sin \theta}{\sin \theta(1 - \sin \theta)} \\ &= \frac{1}{\sin \theta} = \csc \theta \end{aligned}$$

3. Give all solutions to each of the following equations: (10 points each)

(a) $3 \sec^2 x - 4 = 0$ (#11 from 5.3)

$$3 \sec^2 x - 4 = 0$$

$$3 \sec^2 x = 4$$

$$\sec^2 x = \frac{4}{3}$$

$$\sec x = \pm \frac{2}{\sqrt{3}} = \pm \frac{2\sqrt{3}}{3}$$

$$x = \frac{\pi}{6} + 2n\pi$$

$$= -\frac{\pi}{6} + 2n\pi$$

$$= \frac{5\pi}{6} + 2n\pi$$

$$= \frac{7\pi}{6} + 2n\pi,$$

where n is an integer. (There are multiple ways to write these answers.)

(b) $\sin x(\sin x + 1) = 0$ (#13 from 5.3)

$$\sin x(\sin x + 1) = 0$$

$$\sin x = 0$$

$$x = n\pi$$

or $\sin x + 1 = 0$

$$\sin x = -1$$

$$\text{or } x = \frac{3\pi}{2} + 2n\pi,$$

where n is an integer.