Section 5.3, Solving Trigonometric Equations

Homework: 5.3 #7–39 odds

In Section 4.7, we learned about inverse trigonometric functions, which gave only one solution to equations like $\sin x = \frac{1}{2}$. Now, we will focus on finding all angles that solve trigonometric equations.

Examples
Solve the following equations:

1. $2\cos x - 1 = 0$

$$\begin{aligned} 2\cos x - 1 &= 0 \\ 2\cos x &= 1 \\ \cos x &= \frac{1}{2} \\ x &= \frac{\pi}{3} + 2n\pi,\ -\frac{\pi}{3} + 2n\pi \end{aligned}$$

2. $\tan^2 x + \sqrt{3}\tan x = 0$
   We can use techniques from factoring, so:

$$\begin{aligned} \tan^2 x + \sqrt{3}\tan x &= 0 \\ \tan x(\tan x + \sqrt{3}) &= 0 \end{aligned}$$

$$\tan x = 0 \qquad\qquad \tan x = -\sqrt{3}$$

$$x = n\pi \qquad\qquad x = -\frac{\pi}{3} + n\pi$$

3. $4\sin^2 x - 1 = 0$

$$\begin{aligned} 4\sin^2 x - 1 &= 0 \\ (2\sin x - 1)(2\sin x + 1) &= 0 \end{aligned}$$

$$\sin x = \frac{1}{2} \qquad\qquad \sin x = -\frac{1}{2}$$

$$x = \frac{\pi}{6} + 2n\pi,\ \frac{5\pi}{6} + 2n\pi \qquad\qquad x = \frac{7\pi}{6} + 2n\pi,\ \frac{11\pi}{6} + 2n\pi$$

4. $3\sin^2 x = \cos^2 x$
   Here, we have more than one trigonometric function, so we want to change this to having only one trigonometric function. Let's use the identity $\cos^2 x = 1 - \sin^2 x$

$$\begin{aligned} 3\sin^2 x &= \cos^2 x \\ 3\sin^2 x &= 1 - \sin^2 x \\ 4\sin^2 x - 1 &= 0 \end{aligned}$$

   This is now the same as the last problem, so $x = \frac{\pi}{6} + 2n\pi$, $\frac{5\pi}{6} + 2n\pi$, $\frac{7\pi}{6} + 2n\pi$, $\frac{11\pi}{6} + 2n\pi$.

   You could also solve this by first dividing both sides by $\cos^2 x$, which would give you $3\tan^2 x = 1$.

5. $\cot^2 4x = 3$

$$\begin{aligned} \cot^2 4x &= 3 \\ \cot 4x &= \pm\sqrt{3} \\ 4x &= \frac{\pi}{6} + n\pi,\ -\frac{\pi}{6} + n\pi \\ x &= \frac{\pi}{24} + \frac{n\pi}{4},\ -\frac{\pi}{24} + \frac{n\pi}{4} \end{aligned}$$

6. $1 + \tan^2\theta = \sec^2\theta$
All values of θ will satisfy this equation. This will happen whenever your equation is a trigonometric identity.