

## Section 5.2, Verifying Trigonometric Identities

### Homework: 5.2 #1–37 odds

Our goal in this section is to check if two expressions are equal. There are two primary techniques that can be used:

1. Transform one side into the other.
2. Check that both sides simplify to the same expression. This normally works better than the first technique if both sides look complicated.

Here are some helpful hints:

1. Start with the more complicated side.
2. Use trigonometric identities, add fractions, and simplify.
3. If necessary, change trigonometric functions to sines and cosines.
4. Try something! Even if you're not sure where to start, making a minor adjustment may make it look simpler. Also, since there is more than one way to simplify, you might find a good first step by "guessing."

### Examples

Verify each of the following identities:

1.  $\frac{\tan^3 t}{\sec t} = \sin t(\sec^2 t - 1)$

$$\frac{\tan^3 t}{\sec t} = \frac{\tan t}{\sec t} \tan^2 t = \sin t(\sec^2 t - 1)$$

2.  $\frac{1}{\cot \theta} + \cot \theta = \frac{\csc^2 \theta}{\cot \theta}$

$$\frac{1}{\cot \theta} + \cot \theta = \frac{1 + \cot^2 \theta}{\cot \theta} = \frac{\csc^2 \theta}{\cot \theta}$$

3.  $\frac{\csc x - 1}{1 - \sin x} = \csc x$

$$\frac{\csc x - 1}{1 - \sin x} = \frac{\frac{1}{\sin x} - 1}{1 - \sin x} = \frac{\frac{1 - \sin x}{\sin x}}{1 - \sin x} = \frac{1}{\sin x} = \csc x$$

4.  $\sec x - \cos x = \sin x \tan x$

$$\sec x - \cos x = \frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \sin x \tan x$$

5.  $\sin x - \frac{\sin x}{1 - \cot x} = \frac{\sin x \cos x}{\cos x - \sin x}$

$$\sin x - \frac{\sin x}{1 - \cot x} = \frac{\sin x - \cos x - \sin x}{1 - \cot x} = -\frac{\cos x}{1 - \frac{\cos x}{\sin x}} = -\frac{\cos x \sin x}{\sin x - \cos x} = \frac{\sin x \cos x}{\cos x - \sin x}$$

$$6. \frac{\cot x + \cot y}{1 - \cot x \cot y} = \frac{\tan x + \tan y}{\tan x \tan y - 1}$$

$$\frac{\cot x + \cot y}{1 - \cot x \cot y} = \frac{\cot x + \cot y}{1 - \cot x \cot y} \cdot \frac{\tan x \tan y}{\tan x \tan y} = \frac{\tan y + \tan x}{\tan x \tan y - 1}$$

$$7. \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = \frac{1 + \cos \theta}{|\sin \theta|}$$

$$\begin{aligned} \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} &= \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta} \cdot \frac{1 + \cos \theta}{1 + \cos \theta}} = \sqrt{\frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta}} = \sqrt{\frac{(1 + \cos \theta)^2}{\sin^2 \theta}} \\ &= \frac{|1 + \cos \theta|}{|\sin \theta|} = \frac{1 + \cos \theta}{|\sin \theta|} \quad \text{since } 1 + \cos \theta \geq 0 \end{aligned}$$