Section 5.1, Using Fundamental Identities

Homework: 5.1 #5, 11, 27–47 odds, 55, 61

There is a table summarizing properties of trigonometric functions on page 374. Many of them will be useful for these problems.

In this section, we will not be learning any new identities or properties, but rather practicing with the ones that we have already learned.

1 Evaluating Trigonometric Functions

Example
Given that $\sin x = \frac{7}{25}$ and $\cos x < 0$, evaluate all 6 trigonometric functions.

$$\sin x = \frac{7}{25} \qquad \csc x = \frac{25}{7}$$

$$\cos x = -\sqrt{1 - \left(\frac{7}{25}\right)^2} = -\sqrt{\frac{576}{625}} = -\frac{24}{25} \qquad \sec x = -\frac{25}{24}$$

$$\tan x = -\frac{7}{24} \qquad \cot x = -\frac{24}{7}$$

2 Simplifying Expressions

When simplifying expressions, there is more than one way to get to the correct answer. There is also sometimes more than one simplified way to write the correct answer.

If you don't know where to start with a problem, I recommend rewriting the trigonometric functions with sines and cosines only. This can make it easier to see what cancels. If there are multiple terms with fractions, try using a common denominator to write the expression as one fraction to see what cancellation can happen in the numerator.

Examples
Simplify each of the following expressions:

1. $\csc^2 x(1 - \cos^2 x)$

$$\csc^2 x(1 - \cos^2 x) = \csc^2 x \cdot \sin^2 x = 1$$

2. $\dfrac{\cot^2 x}{\csc^2 x}$

$$\frac{\cot^2 x}{\csc^2 x} = \frac{\cos^2 x}{\sin^2 x} \div \frac{1}{\sin^2 x} = \cos^2 x$$

3. $\dfrac{1}{1 + \cot^2 \theta}$

$$\frac{1}{1 + \cot^2 \theta} = \frac{1}{\csc^2 x} = \sin^2 x$$

4. $\csc t \tan t + \sec t$

$$\csc t \tan t + \sec t = \frac{1}{\sin t} \cdot \frac{\sin t}{\cos t} + \frac{1}{\cos t} = 2 \sec t$$

5. $\cos^2 \phi \sec^2 \phi - \cos^2 \phi$

$$\cos^2 \phi \sec^2 \phi - \cos^2 \phi = 1 - \cos^2 \phi = \sin^2 \phi$$

(This one can also be solved by factoring out the $\cos^2 \phi$ first.)

6. $\dfrac{\sin^2 x - 9}{\sin x - 3}$

$$\frac{\sin^2 x - 9}{\sin x - 3} = \frac{(\sin x - 3)(\sin x + 3)}{\sin x - 3} = \sin x + 3$$

7. $\sin^4 \theta - \cos^4 \theta$

$$\begin{aligned}\sin^4 \theta - \cos^4 \theta &= (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta) = \sin^2 \theta - \cos^2 \theta, \quad \text{or} \\ &= 1 - 2\cos^2 \theta, \quad \text{or} \\ &= 2\sin^2 \theta - 1\end{aligned}$$

8. $\sec^3 x - \sec^2 x - \sec x + 1$

$$\begin{aligned}\sec^3 x - \sec^2 x - \sec x + 1 &= \sec^2 x(\sec x - 1) - (\sec x - 1) = (\sec^2 x - 1)(\sec x - 1) \\ &= \tan^2 x(\sec x - 1)\end{aligned}$$

9. $\dfrac{\cos \theta}{1 + \sin \theta} + \dfrac{1 + \sin \theta}{\cos \theta}$

$$\begin{aligned}\frac{\cos \theta}{1 + \sin \theta} + \frac{1 + \sin \theta}{\cos \theta} &= \frac{\cos^2 \theta + (1 + \sin \theta)^2}{(1 + \sin \theta)\cos \theta} = \frac{\cos^2 \theta + 1 + 2\sin \theta + \sin^2 \theta}{(1 + \sin \theta)\cos \theta} \\ &= \frac{2 + 2\sin \theta}{(1 + \sin \theta)\cos \theta} = \frac{2(1 + \sin \theta)}{(1 + \sin \theta)\cos \theta} = 2\sec \theta\end{aligned}$$

10. $\dfrac{1}{\csc \theta + 1} - \dfrac{1}{\csc \theta - 1}$

$$\frac{1}{\csc \theta + 1} - \frac{1}{\csc \theta - 1} = \frac{\csc \theta - 1 - \csc \theta - 1}{(\csc \theta + 1)(\csc \theta - 1)} = \frac{-2}{\csc^2 \theta - 1} = \frac{-2}{\cot^2 \theta} = -2\tan^2 \theta$$