Section 4.6, Graphs of Other Trigonometric Functions

Homework: 4.6 #1–19 odds, 29

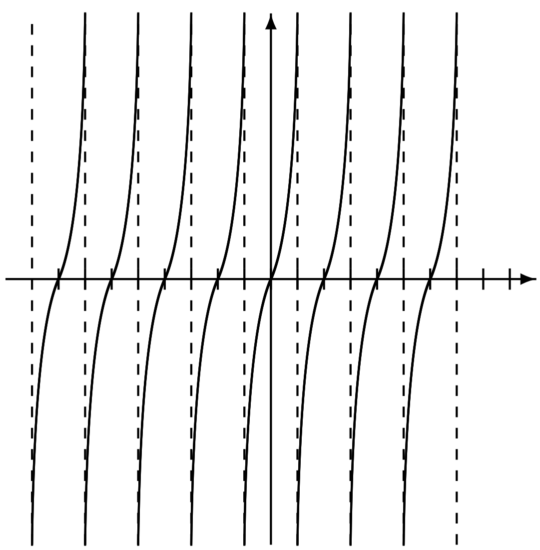
Note: The graphs displayed in these notes are for your reference. Graphs sketched in class and done on quizzes and exams will need to be labeled with more detail.

1 Graph of Tangent

Let $y = \tan x$. Then we know that:

1. There are vertical asymptotes at $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2} \pm n\pi$ (for any integer n). This happens because $\tan x = \frac{\sin x}{\cos x}$, and $\cos x = 0$ at $\frac{\pi}{2} \pm n\pi$ for any integer n.
2. There are x-intercepts at $x = \ldots, -\pi, 0, \pi, 2\pi, \ldots$ since $\sin x = 0$ at any multiple of π.
3. The period of the graph is π. For most trig functions, the period is 2π, but here it's actually smaller, since $\sin(x+\pi) = -\sin x$ and $\cos(x+\pi) = -\cos x$, so $\tan(x+\pi) = \frac{\sin(x+\pi)}{\cos(x+\pi)} = \frac{-\sin x}{-\cos x} = \tan x$.

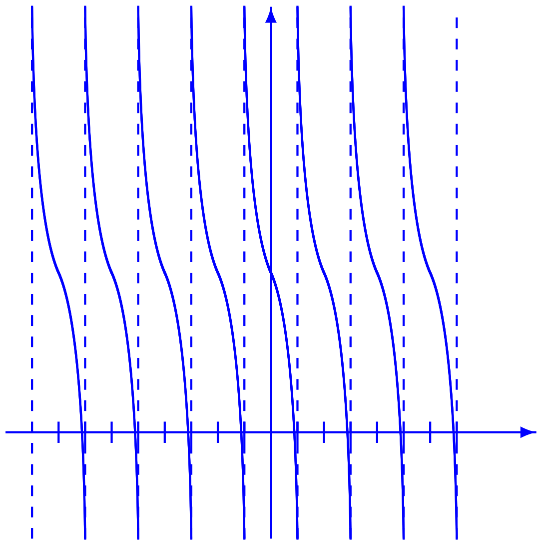
The graph looks like:



(Hash marks on the x-axis are in increments of $\frac{\pi}{2}$.)

Example
Sketch the graph of $y = -2\tan(\frac{x}{2}) + 3$.

The negative sign will "flip" the graph upside-down. The coefficient of 2 will "stretch" the graph from top to bottom. The period is now $\pi/(1/2) = 2\pi$ instead of π. The graph is also shifted up 3.



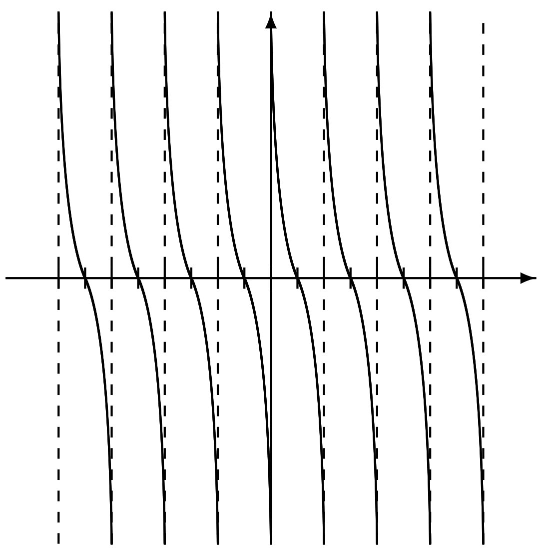
(Hash marks on the x-axis are in increments of π. The y-intercept is now 3.)

2 Graph of Cotangent

Let $y = \cot x$. Then,

1. The graph has vertical asymptotes at all multiples of π (since $\sin x = 0$ at all multiples of π).
2. The graph has x-intercepts at $x = \frac{\pi}{2} + n\pi$ for all integers n (when $\cos x = 0$).
3. The period of this graph is also π.

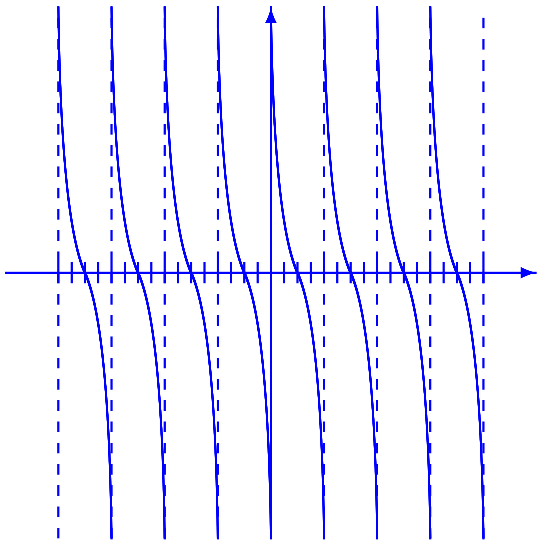
The graph is:



(Hash marks on the x-axis are in increments of $\frac{\pi}{2}$. There is no y-intercept.)

Example
Sketch the graph of $y = 3\cot\frac{x}{4}$.

The 3 will stretch the graph vertically by a factor of 3. The fraction will change the period to 4π instead of π. The graph is:

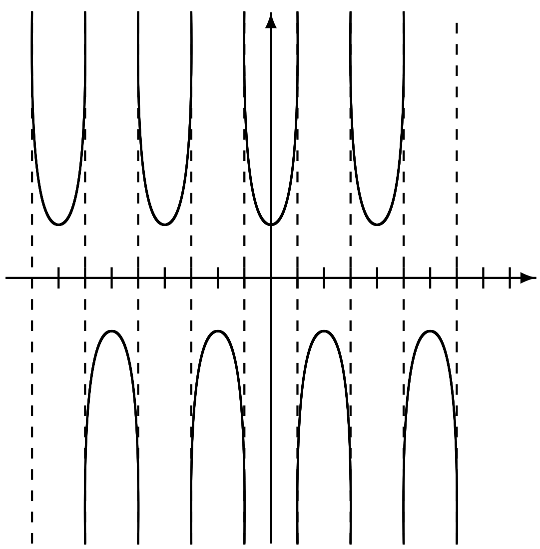


(Hash marks on the x-axis are in increments of π.)

3 Graph of Secant

Let $y = \sec x$. The graph has vertical asymptotes when $x = \frac{\pi}{2} + n\pi$ for every integer n (since $\cos x = 0$ at those x-values). The period of the graph is 2π. There are no x-intercepts. The y-intercept is 1.

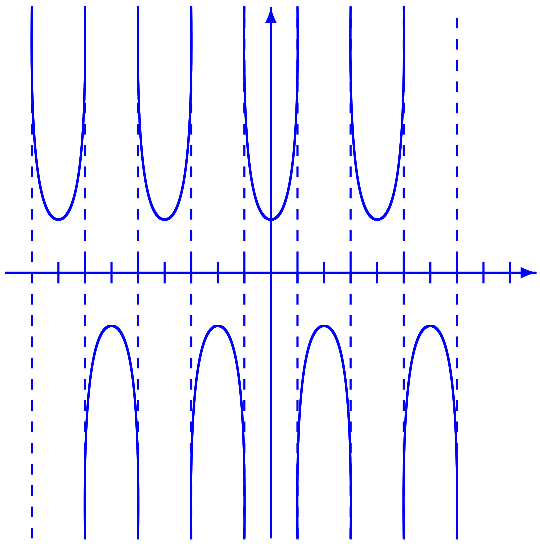
The graph looks like:



(Hash marks on the x-axis are in increments of $\frac{\pi}{2}$.)

Example
Sketch the graph of $y = 3 \sec x$.

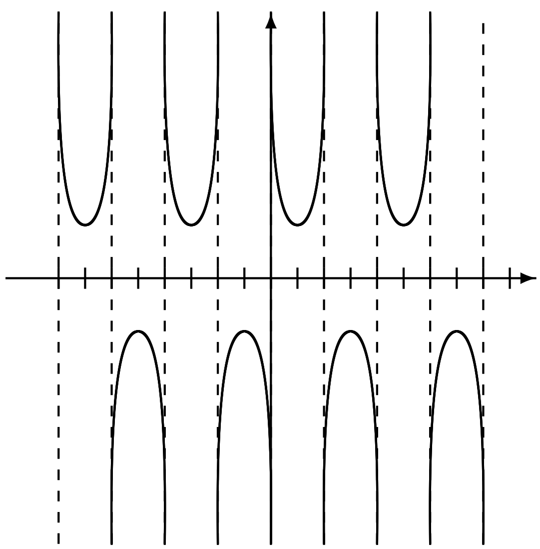
The 3 will "stretch" the graph away from zero, so the values of y will be greater than 3 or less than -3. The graph of this function is:



(Hash marks on the x-axis are in increments of $\frac{\pi}{2}$. The y-intercept is 3.)

4 Graph of Cosecant

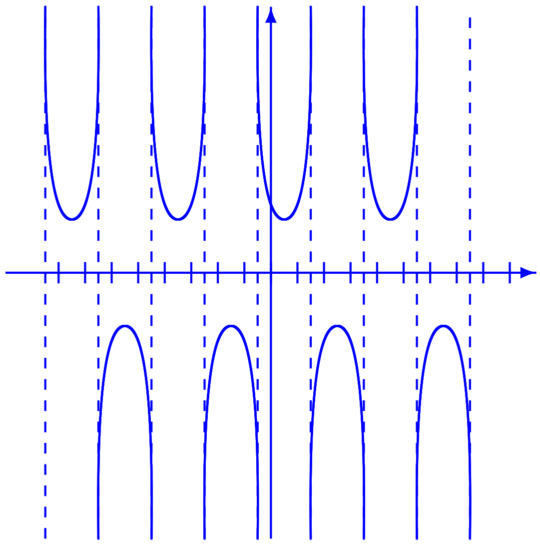
Let $y = \csc x$. The graph has vertical asymptotes at all multiples of π (when $\sin x = 0$). The period of the graph is 2π. There are no x-intercepts. There is no y-intercept.



(Hash marks on the x-axis are in increments of $\frac{\pi}{2}$.)

Example
Sketch the graph of $y = 2\csc(x + \frac{\pi}{4})$.

The graph will now never be between -2 and 2. The $\pi/4$ causes the graph to shift to the left by $\pi/4$.



(The hash marks are in increments of $\pi/2$. The y-intercept is $2\sqrt{2}$.)

5 Dampened Trig Graphs

Example
Sketch the graph of $y = x\cos x$.

As x gets larger, the graph will oscillate more vertically. When x is negative, the cosine pattern will be flipped vertically (just like when the coefficient for cosine is negative). The y-intercept is 0. The x-intercepts are the same as for the cosine function (0 is also an x-intercept here).