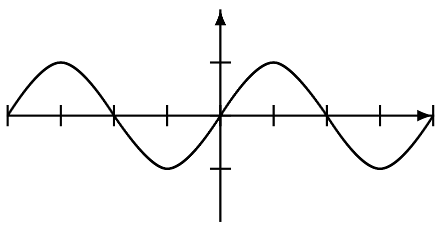
Section 4.5, Graphs of Sine and Cosine Functions

Homework: 4.5 #7–13 odds, 37–49 odds, 53

For our graphs, we will assume that the angle x is given in radians.

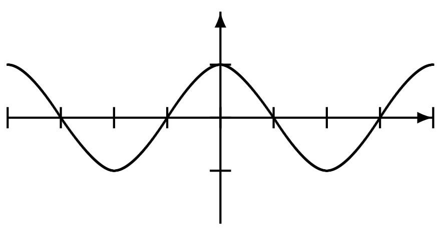
1 Graphs of Sine and Cosine

Let $y = \sin x$. Then its graph is:



(The hash marks on the x-axis are in increments of $\pi/2$.) Also, since sine is 2π-periodic, this pattern repeats.

Let $y = \cos x$. Then its graph is:



(The hash marks on the x-axis are in increments of $\pi/2$ again.) Cosine is also 2π-periodic, this pattern repeats.

The book shows more detailed graphs on page 332.

Our goal for the rest of the section will be to graph functions of the form

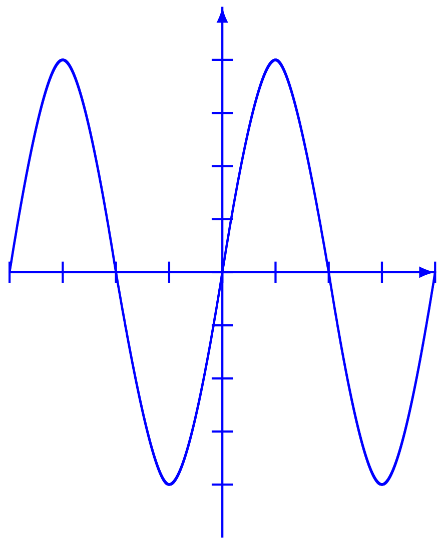
$$y = a\sin(bx - c) + d \text{ and}$$
$$y = a\cos(bx - c) + d, \text{ where } a,\ b,\ c, \text{ and } d \text{ are real numbers}$$

We will focus on one additional number at a time.

If $y = a\sin x$ or $y = a\cos x$, we say that $|a|$ is the **amplitude** of y. Now, instead of having the graph go from $y = -1$ to $y = 1$, it will go from $y = -a$ to $y = a$.

Examples

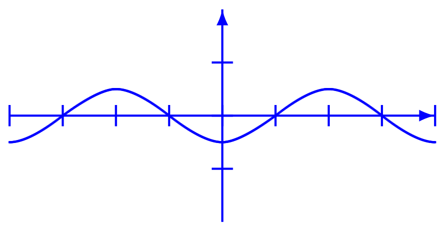
1. Sketch the graph of $y = 4\sin x$.
   The amplitude of the function is 4. The graph is:



(The hash marks on the x-axis are in increments of $\pi/2$.)

2. Sketch the graph of $y = -\frac{1}{2}\cos x$.

   The amplitude of the function is $\frac{1}{2}$. The negative sign will "flip" the graph upside down.



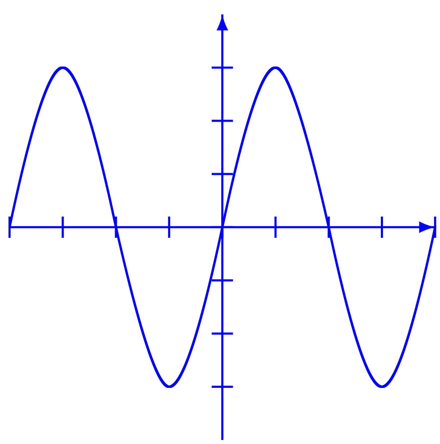
(The hash marks on the x-axis are in increments of $\pi/2$.)

Now, we will focus on functions of the form $y = a\sin(bx)$ and $y = a\cos(bx)$. We will assume that $b > 0$ (If b is negative, we can use that sine is an odd function and that cosine is an even function to rewrite it for $|b|$.)

The **period** of the graph changes to $\frac{2\pi}{b}$.

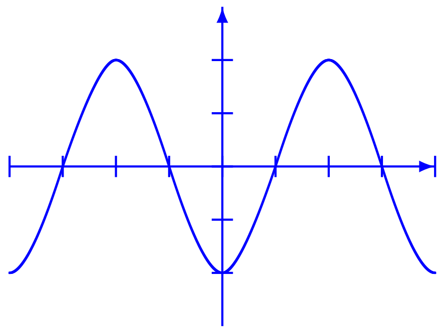
Examples

1. Sketch the graph of $y = 3\sin 2x$.
   The amplitude is 3 and the period is $\frac{2\pi}{2} = \pi$.



(The hash marks on the x-axis are in increments of $\pi/4$.)

2. Sketch the graph of $y = -2\cos\frac{x}{2}$.
   The amplitude of the function is 2. The negative sign will "flip" the graph upside down. The period of the graph is $\frac{2\pi}{1/2} = 4\pi$.

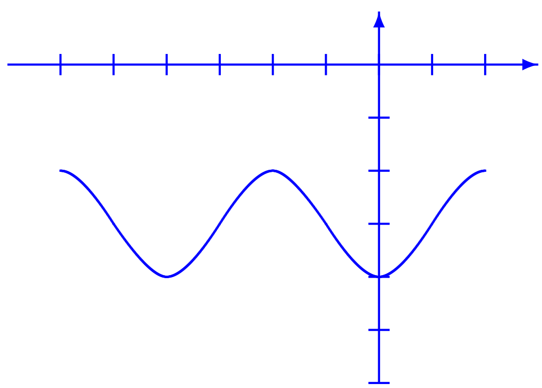


(The hash marks on the x-axis are in increments of π.)

Now, we will focus on horizontal and vertical shifts, which come from including the c and d in $y = a\sin(bx - c) + d$ and $y = a\cos(bx - c) + d$. (The d creates the vertical shift, and the c creates the horizontal shift.)

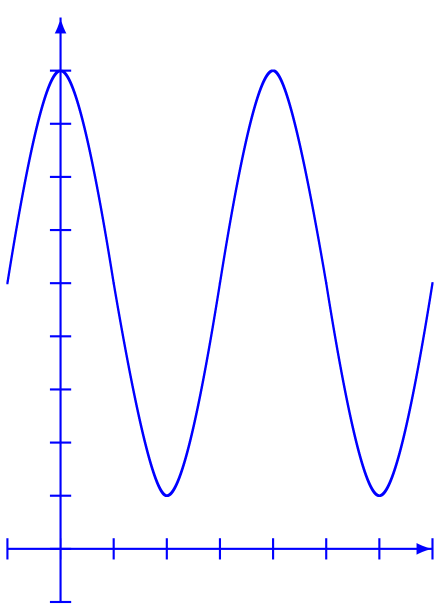
Examples

1. Sketch the graph of $y = \cos(x + \pi) - 3$.
   The amplitude is 1 and the period is 2π. The -3 will shift the graph down 3 units. The $+\pi$ will cause the "pattern" for cosine to go from $-\pi$ to π instead of from 0 to 2π (we get these numbers by solving $x + \pi = 0$ and $x + \pi = 2\pi$.).

   (The hash marks on the x-axis are in increments of $\pi/2$.)

2. Sketch the graph of $y = -4\sin(4x - \frac{\pi}{2}) + 5$.
   The amplitude is 4 and the period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The graph is flipped upside-down (due to the -4 before the sine). It is also shifted up 5, and the cycle that normally goes from 0 to 2π is repeated from $\pi/8$ to $5\pi/8$ (we get these numbers by solving $4x - \frac{\pi}{2} = 0$ and $4x - \frac{\pi}{2} = 2\pi$).

   (The hash marks on the x-axis are in increments of $\pi/8$.)