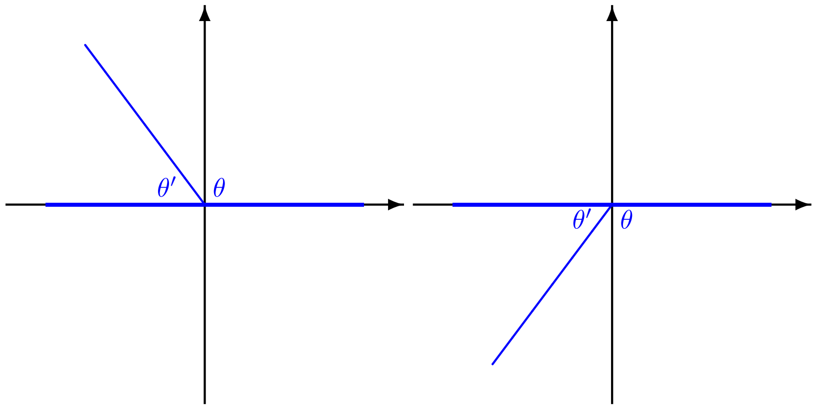
Section 4.4, Trigonometric Functions of Any Angle

Homework: 4.4 #1–23 odds, 29–57 odds

In this section, we will learn how to use our knowledge of trigonometric functions acute angles to calculate trigonometric functions of angles in other quadrants. As a result, some of the homework problems are based on material from previous sections.

1 Using Reference Angles

If θ is an angle in standard position, its reference angle, θ', is the acute angle formed by the terminal side of θ and the horizontal axis. ($0 \le \theta' \le \frac{\pi}{2}$).



After finding a reference angle, we can use that

$$
\begin{aligned}
\left|\sin\theta\right| &= \sin\theta' \\
\left|\cos\theta\right| &= \cos\theta' \\
\left|\tan\theta\right| &= \tan\theta' \\
\left|\csc\theta\right| &= \csc\theta' \\
\left|\sec\theta\right| &= \sec\theta' \\
\left|\cot\theta\right| &= \cot\theta'
\end{aligned}
$$

We can then use our knowledge of the sign ($\pm$) of the trigonometric functions based on the quadrant in which the angle lies (from §4.2) to get our final answer.

Examples

1. Find the reference angles and then evaluate sine, cosine, and tangent for each of the following:

   (a) $\theta = \dfrac{7\pi}{6}$

   The reference angle is $\theta' = \frac{\pi}{6}$ and θ is in quadrant 3 (where sine and cosine are negative and tangent is positive), so

$$
\begin{aligned}
\sin\frac{7\pi}{6} &= -\frac{1}{2} \\
\cos\frac{7\pi}{6} &= -\frac{\sqrt{3}}{2} \\
\tan\frac{7\pi}{6} &= \frac{\sqrt{3}}{3}
\end{aligned}
$$

(b) $\theta = \dfrac{7\pi}{4}$
The reference angle is $\theta' = \frac{\pi}{4}$ and θ is in quadrant 4 (where sine and tangent are negative and cosine is positive), so

$$\sin\frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$$
$$\cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\tan\frac{7\pi}{4} = -1$$

(c) $\theta = \dfrac{2\pi}{3}$
The reference angle is $\theta' = \frac{\pi}{3}$ and θ is in quadrant 2 (where cosine and tangent are negative and sine is positive), so

$$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$
$$\cos\frac{2\pi}{3} = -\frac{1}{2}$$
$$\tan\frac{2\pi}{3} = -\sqrt{3}$$

2. If θ is in Quadrant 3 and $\cos\theta = -\frac{4}{5}$, calculate all 6 of the trigonometric functions for θ. (#16)
In quadrant 3, sine, cosine, cosecant, and secant are negative, and tangent and cotangent are positive. We can use the Pythagorean Theorem to see that $\sin\theta = -\frac{3}{5}$, and:

$$\sin\theta = -\frac{3}{5} \qquad \csc\theta = -\frac{5}{3}$$
$$\cos\theta = -\frac{4}{5} \qquad \sec\theta = -\frac{5}{4}$$
$$\tan\theta = \frac{3}{4} \qquad \cot\theta = \frac{4}{3}$$

3. Given that $\cos\theta = \frac{8}{17}$ and $\tan\theta < 0$, calculate all 6 of the trigonometric functions for θ. (#18)
Since cosine is positive and tangent is negative, we know that sine is negative (We could reason instead that θ is in quadrant 4). Also, by the Pythagorean Theorem, we know that $\sin\theta = -\frac{15}{17}$

$$\sin\theta = -\frac{15}{17} \qquad \csc\theta = -\frac{17}{15}$$
$$\cos\theta = \frac{8}{17} \qquad \sec\theta = \frac{17}{8}$$
$$\tan\theta = -\frac{15}{8} \qquad \cot\theta = -\frac{8}{15}$$

4. Given that $\sin\theta = 0$ and $\sec\theta = -1$, calculate all 6 of the trigonometric functions for θ. (#22)

$$\sin\theta = 0 \qquad \csc\theta = \text{undefined}$$
$$\cos\theta = -1 \qquad \sec\theta = -1$$
$$\tan\theta = 0 \qquad \cot\theta = \text{undefined}$$