Section 4.1, Radian and Degree Measure

Homework: 4.1 #1–69 odds

1 Drawing Angles in Standard Position

In standard position, the **initial side** of the angle lies along the positive x-axis, the vertex of the angle is the origin, $(0,0)$. For positive angles, you move counterclockwise to get to the **terminal side**. For negative angles, you move clockwise.

There are 2 ways of measuring angles: Degrees and Radians.

2 Degrees

A full revolution corresponds to $360°$. There is a good diagram on page 285 of the book.

Two angles are **coterminal** if they have the same initial and terminal sides (i.e., they are different angles that end with the same terminal side). They will differ by a multiple of $360°$.

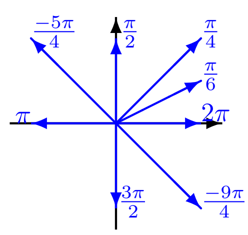
Example
Find 2 coterminal angles for $72°$.
Adding $360°$, we get $432°$. Subtracting $360°$, we get $-288°$. (Note: there are infinitely many possible solutions)

3 Radians

The measure of an angle θ in radians is the ratio of the length of the arc s the angle creates to the radius r of the circle, $\theta = s/r$.

As a result, there are 2π radians in one rotation (since $C = 2\pi r$ for the circumference of a circle).

Example
Graph each of the following angles:
$\pi/4$, π, 2π, $-9\pi/4$, $3\pi/2$, $\pi/6$, $\pi/2$, $-5\pi/4$



4 Converting between Degrees and Radians

Since one rotation is $360°$ in degrees and 2π in radians, we can see that $1 = \frac{360°}{2\pi} = \frac{180°}{\pi}$ can be used to convert from radians to degrees. To convert from degrees to radians, we can multiply by $\frac{\pi}{180°}$.

Examples

1. Convert the following angles to radians:

(a) $240°$

$$240° \cdot \frac{\pi}{180°} = \frac{4}{3}\pi$$

(b) $135°$

$$135° \cdot \frac{\pi}{180°} = \frac{3}{4}\pi$$

(c) $-60°$

$$-60° \cdot \frac{\pi}{180°} = -\frac{\pi}{3}$$

2. Convert the following angles to degrees:

   (a) $2\pi/3$

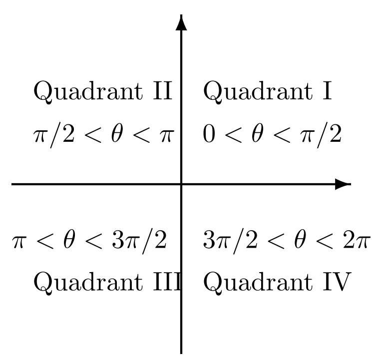
$$\frac{2\pi}{3} \cdot \frac{180°}{\pi} = 120°$$

   (b) $-7\pi/6$

$$-\frac{7\pi}{6} \cdot \frac{180°}{\pi} = -210°$$

5 Quadrants

The four quadrants are identified the same way they are with graphing “normal” points.



These also hold for all coterminal angles. The “boundaries,” (0, $\pi/2$, π, $3\pi/2$, etc.) are not in any quadrant. Rather, they are normally considered to be between quadrants.

Example
Determine the quadrant for each of the following angles:

1. $\pi/4$
   This is in quadrant I.
2. $-25\pi/12$
   This angle is coterminal with $-\pi/12$, so it is in Quadrant IV.
3. 4π
   This is not in any quadrant, since it is an angle on the boundary of two quadrants.

6 Complementary and Supplementary Angles

Two positive angles are **complementary** if their sum is $\pi/2$ ($90°$).

Two positive angles are **supplementary** if their sum is π ($180°$).

Examples

1. Find the complement and supplement of $\pi/6$.
   Complement: $\frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$.
   Supplement: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

2. Find the complement and supplement of $135°$.
   Complement: $90° - 135° = -45° < 0$, so there is no complement.
   Supplement: $180° - 135° = 45°$.