

Section 10.8, Graphs of Polar Coordinates

Homework: 10.8 #21–29 odds

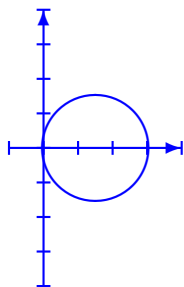
Example

Sketch the graph of $r = 3 \cos \theta$.

Similarly to graphing in rectangular coordinates, we can use several values of θ to see what value is given for r , then graph the function:

θ	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π	$7\pi/6$	$3\pi/2$	$11\pi/6$	2π
r	3	$3\sqrt{3}/2$	$3/2$	0	$-3/2$	$-3\sqrt{3}/2$	-3	$-3\sqrt{3}/2$	0	$3\sqrt{3}/2$	3

Since cosine is 2π -periodic, we only need to consider θ between 0 and 2π (or any other interval with length 2π). Graphing the information that we found, we get the graph:



Note: This gets traced twice when graphing between $\theta = 0$ and 2π .

The problem with this method is that it is fairly tedious. Also, there is a good chance that a point where something “interesting” happens may be missed. One thing that can help us to avoid using this method is to find the maximum and minimum value that the radius can take, and at what angle it happens. We can also use symmetry.

1 Symmetry in Graphing with Polar Coordinates

- If replacing (r, θ) by $(r, -\theta)$ gives an equivalent equation, the graph is symmetric with respect to the polar axis (the horizontal axis). For example, if $r = \cos \theta$ and we replace θ by $-\theta$, we get $r = \cos(-\theta) = \cos \theta$ since cosine is an even function. Since this is what we started with, we know that the graph is symmetric with respect to the polar axis.
- If replacing (r, θ) by $(r, \pi - \theta)$ or $(-r, -\theta)$ gives an equivalent equation, the graph is symmetric with respect to the line $\theta = \pi/2$ (the vertical axis). For example, if $r = \sin \theta$, replacing r by $-r$ and θ by $-\theta$ gives $-r = \sin(-\theta) = -\sin \theta$. After we cancel out the negative signs, this is exactly what we started with, so we know that the graph of $r = \sin \theta$ is symmetric with respect to the line $\theta = \pi/2$.
- If replacing (r, θ) by $(r, \pi + \theta)$ or $(-r, \theta)$ gives an equivalent equation, the graph is symmetric with respect to the pole (origin). For example, $r = 5$ and $\theta = \pi/4$ satisfy this criterion.

Examples

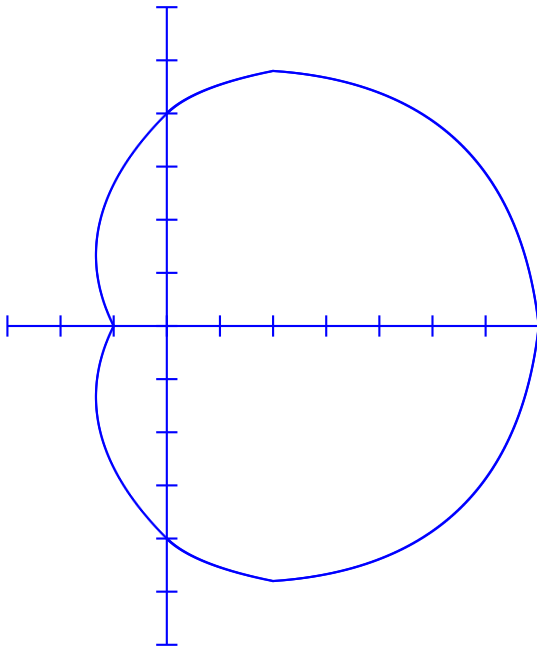
Graph each of the following functions:

1. $r = 4 + 3 \cos \theta$

Since r is a function of $\cos \theta$, we know that it is symmetric with respect to the polar axis. This means that we can focus on the part of the graph from $\theta = 0$ to $\theta = \pi$ and use the mirror image to graph from $\theta = \pi$ to $\theta = 2\pi$.

The maximum value of r is 7 and occurs when $\theta = 0$ and $\theta = 2\pi$. The minimum value of r is 1 and occurs when $\theta = \pi$. There are no zeroes for r since $3 \cos \theta \neq -4$ for any value of θ .

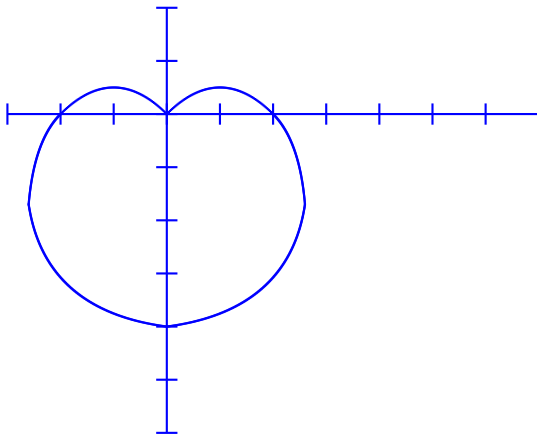
If this is not enough information to use, we can now use information from other points. At $\theta = \pi/2$, $r = 4$. Then, the graph looks like:



2. $r = 2(1 - \sin \theta)$

First, note that this is symmetric with respect to the line $\theta = \pi/2$. This means that we can graph the right (or left) half of the graph, then use reflection to sketch the second half.

The maximum radius is 4 and occurs when $\theta = 3\pi/2$. The radius is zero at $\theta = \pi/2$. To further help us, we can find another value of the radius. When $\theta = 0$ and π , $r = 2$.



3. $r = \theta$

This graph is a spiral shape, starting at the origin, going counterclockwise.

There are many other graphs shown on page 789.