

Section 10.7, Polar Coordinates

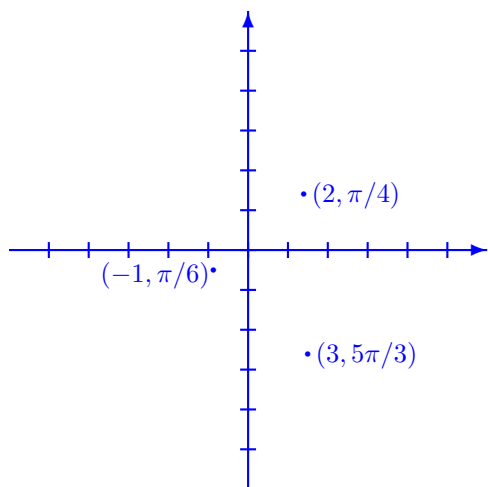
Homework: 10.7 #1–7 odds, 17–25 odds, 35–61 odds

Our goal for this section is to change from the rectangular coordinate system with points (x, y) to **Polar Coordinates**, (r, θ) , where r is the radius (the distance of the point from the origin), and θ is the angle that a ray from the origin to the point would make.

Example

Graph the following points on the Polar Coordinate System:

$$\left(2, \frac{\pi}{4}\right), \quad \left(3, \frac{5\pi}{3}\right), \quad \left(-1, \frac{\pi}{6}\right)$$



As we see, finding the angles is a lot like plotting angles on the unit circle, but the radius doesn't need to be 1. Also, there are multiple ways to represent points in Polar Coordinates, since we can use coterminal angles. We can also use a negative radius if we change the angle appropriately.

1 Converting between Rectangular and Polar Coordinates

To convert between rectangular and polar coordinates, we can use the following formulas:

$$x = r \cos \theta$$

$$\tan \theta = \frac{y}{x}$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2 \text{ or } r = \sqrt{x^2 + y^2}$$

Examples

1. Convert $\left(3, \frac{\pi}{2}\right)$ to rectangular coordinates.

We can use $x = r \cos \theta$ and $y = r \sin \theta$ to get values for x and y :

$$x = r \cos \theta = 3 \cos \frac{\pi}{2} = 0$$

$$y = r \sin \theta = 3 \sin \frac{\pi}{2} = 3,$$

so the point in rectangular coordinates is $(0, 3)$.

2. Convert $(-1, -1)$ to polar coordinates.

First, we can find r :

$$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

To find θ , we can use that the point is in Quadrant III along with $\tan \theta = y/x$ to get

$$\begin{aligned}\tan \theta &= \frac{-1}{-1} = 1 \\ \theta &= \frac{5\pi}{4},\end{aligned}$$

so the point is $(\sqrt{2}, \frac{5\pi}{4})$.

3. Convert the following rectangular equations to polar form:

(a) $x^2 + y^2 = 25$

Our goal for these examples will be to replace all x 's and y 's, and replace them by expressions involving r 's and θ 's. We know that $x^2 + y^2 = r^2$, so:

$$\begin{aligned}x^2 + y^2 &= 25 \\ r^2 &= 25 \\ r &= 5\end{aligned}$$

We don't actually need to allow for $r = -5$ in this case, since this holds for all values of θ . For example, the point $(-5, \frac{\pi}{2})$ satisfies the equation $r = -5$, but $(-5, \frac{\pi}{2}) = (5, \frac{3\pi}{2})$, so the point can be represented by another point that satisfies $r = 5$.

(b) $2x = 3y$

We can replace x and y by $r \cos \theta$ and $r \sin \theta$, respectively, so

$$\begin{aligned}2x &= 3y \\ 2r \cos \theta &= 3r \sin \theta \\ \frac{2}{3} &= \frac{r \sin \theta}{r \cos \theta} \\ \tan \theta &= \frac{2}{3}\end{aligned}$$

(c) $y = 5$

$$\begin{aligned}y &= 5 \\ r \sin \theta &= 5\end{aligned}$$

4. Convert the following polar equations to rectangular form:

(a) $r = 8$

$$\begin{aligned}r &= 8 \\ \sqrt{x^2 + y^2} &= 8 \text{ or } x^2 + y^2 = 64\end{aligned}$$

(b) $\theta = \frac{\pi}{6}$

We can start by taking the tangent of both sides:

$$\begin{aligned}\tan \theta &= \tan \frac{\pi}{6} \\ \frac{y}{x} &= \frac{\sqrt{3}}{3} \\ y &= \frac{\sqrt{3}}{3}x\end{aligned}$$

(c) $r = 2 \sin \theta$

There are multiple ways to start the problem. I will start by multiplying both sides of the equation by r . Then,

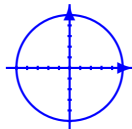
$$r^2 = 2r \sin \theta$$

$$x^2 + y^2 = 2y$$

5. Graph the following equations:

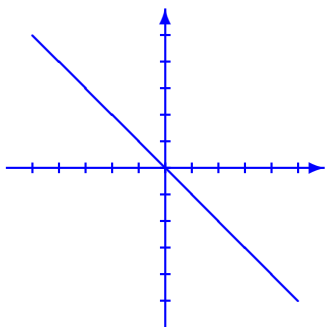
(a) $r = 5$

This is a circle of radius 5.



(b) $\theta = \frac{3\pi}{4}$

This is a line through the origin with a slope of -1 . You can see this by taking the tangent of both sides, then using that $\tan \theta = y/x$, and then solving for y .



Note: The next section will be devoted to graphing in Polar Coordinates.