Quiz 8 Key

Math 1220–7

November 30, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. No calculators are allowed on this quiz.

1. Find the Maclaurin Series for $f(x) = e^{-x}$. Include terms through at least the x^3 term. (25 points)

   We will need to find the derivatives at $x = 0$:

$$f(x) = e^{-x} \qquad f(0) = 1$$
$$f'(x) = -e^{-x} \qquad f'(0) = -1$$
$$f''(x) = e^{-x} \qquad f''(0) = 1$$
$$f'''(x) = -e^{-x} \qquad f'''(0) = -1$$

   Then, the Maclaurin Series is

$$\begin{aligned} f(x) &= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots \\ &= 1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \cdots \end{aligned}$$

2. Find the Taylor Polynomial in $\left(x - \frac{\pi}{3}\right)$ of order 2 for the function $f(x) = \cos x$. (25 points)

   We will need to find the derivatives at $x = \frac{\pi}{3}$:

$$f(x) = \cos x \qquad f(\pi/3) = \frac{1}{2}$$
$$f'(x) = -\sin x \qquad f'(\pi/3) = -\frac{\sqrt{3}}{2}$$
$$f''(x) = -\cos x \qquad f''(\pi/3) = -\frac{1}{2}$$

   Then, the Taylor polynomial of order 2 is

$$\begin{aligned} f(x) &= f(\pi/3) + f'(\pi/3)\left(x - \frac{\pi}{3}\right) + \frac{f''(\pi/3)}{2!}\left(x - \frac{\pi}{3}\right)^2 \\ &= \frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) - \frac{1}{4}\left(x - \frac{\pi}{3}\right)^2 \end{aligned}$$

3. Find a good (upper) bound for $\left|\dfrac{c+2}{c+3}\right|$, where $c \in [0,2]$. (10 points)

$$\left|\frac{c+2}{c+3}\right| \leq \frac{2+2}{c+3} \leq \frac{4}{0+3} = \frac{4}{3}$$

Note: There are other ways to do this problem, such as taking the derivative to find the critical values.