Quiz 7 Key
Math 1220–7
November 9, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. No calculators are allowed on this quiz. Each part of each question is worth 15 points.

1. Determine whether each of the following series converges or diverges. Be sure to justify each answer with an appropriate test.

   (a) $\displaystyle\sum_{n=1}^{\infty} ke^{-3k^2}$ (#11 from 9.3)

   Using the integral test, we get:

   $$\int_1^{\infty} xe^{-3x^2}\,dx = -\frac{1}{6}\,e^{-3x^2}\Big|_1^{\infty} = -\frac{1}{6}\cdot 0 + \frac{1}{6}\,e^{-3} = \frac{1}{6e^3}$$

   Since this integral converges, the sum also converges.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{8^n}{n!}$ (#5 from 9.4)

   Using the ratio test, we see that

   $$\frac{a_{n+1}}{a_n} = \frac{8^{n+1}/(n+1)!}{8^n/n!} = \frac{8}{n+1}$$

   Taking the limit as $n \to \infty$, this goes to 0. Since $0 < 1$, the sum converges.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+200}$ (#11 from 9.4)

Since $\displaystyle\lim_{n\to\infty} \frac{n}{n+200} = 1 \neq 0$, the sum diverges by the n^{th}-term test for divergence.

2. Determine whether the following series is absolutely convergent, conditionally convergent, or divergent: (#21 from 9.5)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$$

Since this is an alternating series with decreasing terms, and $\displaystyle\lim_{n\to\infty} \frac{n}{n^2+1} = 0$, the series converges. However, the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ diverges by the Limit Comparison Test (we compare $\frac{n}{n^2+1}$ to $1/n$), so the series is conditionally convergent.